Master-Slave Synchronous Position Control for Precision Stages
Based on Multirate Control and Dead-time Compensation

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Abstract—A synchronous position control system based on master-slave method is often required in industrial equipment, for example, NC machine tools, exposure systems, and so on. The authors’ research group has achieved precise positioning whose error tolerance is in sub-micrometer range for a large-scaled high-precision stage with perfect tracking control (PTC). In this paper, we propose a novel synchronous position control system for a pair of precision stages based on multirate control and dead-time compensation. Finally, simulations and experiments with experimental precision stages are performed to show the advantages of the proposed control system.

I. INTRODUCTION

The synchronous position control method for multi servo systems is divided into two main classes. One is the method in which dynamic and response characteristics of each servo system are matched and the references of each servo system are put out in synchronization. However, it is difficult to match the control characteristic of each servo system perfectly. Moreover, it is impossible to synchronize positions of each system precisely. The other is master-slave method in which the output of one servo system with slow response characteristic (Master) is the reference of the other servo system with fast response characteristic (Slave). In this method, designing 2-degrees of freedom control system for Slave, the synchronous position control system is achieved more precisely [1].

A synchronous position control system based on master-slave method is often required in industrial equipment, for example, NC machine tools, exposure systems, and so on [2]. The authors’ research group has achieved precise positioning whose error tolerance is in sub-micrometer range for a large-scaled high-precision stage with perfect tracking control (PTC) [3], [4] and [5].

In this paper, a synchronous position control system for two precision stages is proposed. Then, it is assumed that PTC which is explained in next section has been designed for each precision stage. The control system can synchronize two nominal precision stages perfectly. However, there are not only disturbances but also dead-times in the system. These obstacles influence the synchronization accuracy. The proposed synchronous position control system can be more resistant to the disturbance of master stage and plant vari-

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Fig. 1. Perfect tracking control system.

Fig. 2. Multirate sampling period.

II. PERFECT TRACKING CONTROL [3]

Perfect tracking control (PTC) consists of the 2-DOF control system as shown in Fig. 1. There exist sampling periods $T_r$, $T_u$, and $T_u$ which represent the periods of the reference signal $r(t)$, the output $y(t)$, and the input $u(t)$, respectively.

PTC applies the multirate feedforward control in which the control input $u(t)$ is changed $n$ times during one sampling period $T_r$ of reference input $r(t)$ as shown in Fig. 2. Here, $n$ is the plant order. $H_M$ in Fig. 1 is the multirate holder which outputs the input $u[i] = [u_1[k], \ldots, u_n[k]]^T$ (generated by the long sampling period $T_u$) on the short sampling period $T_r$. MrFFC in Fig. 1 is the multirate feedforward controller which is the stable inverse system of the plant as the references are state variables $x_{d}[k+n]$. Therefore, the tracking error of plant state becomes perfectly zero at every $T_r$ by the multirate feedforward controller if the plant is nominal.

Moreover, the feedback control $C_2[z]$ suppresses the error between the output $y[k]$ and the nominal output $y_0[k]$ to assure robustness only when disturbances or plant variations exist.
C. Ver. 3: Real Position FB + PTC

In Fig. 5, above two methods are combined. The system can achieve the synchronization between Master and Slave perfectly if the plant is nominal. Moreover, the system can be resistant to the disturbance of master stage and plant variations because the real position of Master is fed back. Though the system is adapted for an ideal system without dead-times, dead-times of the system influence the tracking accuracy in a real system.

D. Ver. 4: Observer which compensates dead-times (Proposed)

Therefore, we proposed the system with an observer considering dead-times of the system as shown in Fig. 6. Since the state variables (position, velocity, ···) $n_s$ sample ahead of Master are given to the references of PTC per $n_s$ sampling period in Slave, the system can be more resistant to the disturbance of master stage and plant variations. The key point of the control system is how to estimate the state variables $n_s$ sample ahead of Master with the observer.

IV. CONSIDERATION OF DEAD-TIME

The class of the problem is assumed as $T_{um} = T_{us} = T_{ym} = T_{ys} = T_u = T_y$ easily though many various cases are considered about sampling periods of Master and Slave. Dead-times which occur in Master and Slave are discussed.
Dead-times which occur in the plants are defined as $T_{dim}$ and $T_{dis}$. Dead-times which occur in the sensors are defined as $T_{dom}$ and $T_{dos}$ as a delay exists in the output. Fig. 7 shows the plant with dead-times. Here, the state $x_m$ of Master is defined as a controllable canonical form. The continuous time state equation of (3) per $T_u$ is represented as

$$\dot{x}(t) = A_c x(t) + b_c u(t)$$

and the real position $y_m(t)$ of Master is needed to synchronize the real position $y_s(t)$ of Slave and the real position $y_m(t)$ of Master, the estimated state $\hat{x}_m$ has to be stepped for $n_s T_u + T_{dis} - T_{dim}$ periods moreover.

A. Definition of State Equations of Plants

The continuous time state equation of the plant of Master except dead-times is represented as

$$\begin{align*}
\dot{x}_{pm}(t) &= A_{cpm} x_{pm}(t) + b_{cpm}(u_m(t) - d_m(t)) \\
y_m(t) &= c_{cpm} x_{pm}(t)
\end{align*}$$

as a controllable canonical form. The continuous time state equation of the disturbance of Master is represented as

$$\begin{align*}
\dot{d}_m(t) &= A_{cmd} d_m(t) \\
\dot{d}_m(t) &= c_{cmd} d_m(t)
\end{align*}$$

The augmented continuous time state equation of Master can be represented as

$$\begin{align*}
\dot{x}_m(t) &= A_c x_m(t) + b_c u_m(t) \\
y_m(t) &= c_c x_m(t)
\end{align*}$$

$$A_c = \begin{bmatrix}
A_{cpm} & -b_{cpm} c_{cmd} & b_{cpm} \\
0 & A_{cmd} & 0 \\
0 & 0 & 0
\end{bmatrix}$$

where $x_m = [x_{pm}, x_{dm}]^T$. Then, the augmented discrete time state equation of (3) per $T_u$ is represented as

$$\begin{align*}
x_m[k + 1] &= A_{zm} x_m[k] + b_{zm} u_m[k] \\
y_m[k] &= c_{zm} x_m[k]
\end{align*}$$

In Slave, these equations are defined similarly as the subscript $m$ is changed into $s$.

B. Estimation of States of Master

The dead-time of Master summed up in the output of Master is defined as

$$T_{dm} = T_{dim} + T_{dom}.$$  \hfill (6)

Thus, the output of Master delays for the dead-time $T_{dm}$ as Fig. 8.

Here, the state $x_m(k T_y + T_{dim})$ of Master at the time $k T_y$ is estimated as the state $\hat{x}_m[k]$ by an observer.

1) In the case of $T_{dm} = 0$: The state $x_m[k]$ of the plant of Master can be estimated by the general observer as

$$\hat{x}_m[k] = A_{zm} \hat{x}_m[k-1] + b_{zm} u_m[k-1] + H(y_m[k-1] - c_{zm} \hat{x}_m[k-1]),$$

$$y_m[k] = x_m[k] - \hat{x}_m[k]$$

$$e_m[k] = (A_{zm} - H c_{zm}) e_m[k-1].$$

2) In the case of $(n_{dm} - 1) T_y < T_{dm} \leq n_{dm} T_y$ ($n_{dm} = 1, 2, \ldots$): At the time $k T_u$, the available output signal $y_m[k]$ is represented as

$$y_m[k] = c_{dm} x_m[k - n_{dm}] + d_{dm} u_m[k - n_{dm}],$$

$$c_{dm} = c_{zm} e^{A_{zm} T_y - T_{dim}},$$

$$d_{dm} = \int_0^{n_{dm} T_y - T_{dim}} c_{zm} e^{A_{zm} T_y - T_{dim}} dT.$$  \hfill (11)

The state $x_m[k]$ of the plant of Master can be estimated by the equation as follows:

$$\hat{x}_m[k] = A_{zm}^{n_{dm}} \hat{x}_m[k - n_{dm}]$$

$$+ [A_{zm}^{n_{dm} - 1} b_{zm}, A_{zm}^{n_{dm} - 2} b_{zm}, \ldots, b_{zm}]\begin{bmatrix} u_m[k - n_{dm}] \\
\vdots \\
\vdots \\
u_m[k - 1]
\end{bmatrix}$$

$$- d_{dm} u_m[k - n_{dm}],$$

$$e_m[k] = (A_{zm}^{n_{dm}} - A_{zm}^{n_{dm} - 1} H c_{dm}) e_m[k - n_{dm}].$$  \hfill (13)

C. Generation of References for Slave

Since the state $\hat{x}_m[k]$ of Master at $k T_y$ has been estimated, the state $\hat{x}_m[i + N + \sigma]$ which is stepped $n_s T_y + T_{dis} - T_{dim}$ periods ahead is to be estimated per $n_s$ periods. Here, $[i] = (i \cdot n_s T_y), N = 0, 1, \ldots$ and $0 \leq \sigma < 1.$

In the case that $n_s T_y + T_{dis} - T_{dim} < 0$, $\hat{x}_m[k]$ has only to be delayed for this period.
where,
\[
\begin{align*}
\{ \mathbf{b}_{dm} & = \frac{a_{zm}^{-1} b_{zm}}{z^j} (j \leq N) \\
\mathbf{b}_{dm}(N+1) & = \int_0^N \mathbf{A}_m(z) d \tau \\
u_m & = u_m[k] \\
u_m & = u_m[k+j-1] (2 \leq j \leq N+1)
\end{align*}
\]  
(18)  
(19)

Here, \( u_m \) represents a nominal input calculated by the multirate feedback controller previously.

Thus, the elements of the state \( \hat{x}_m[i+N+\sigma] \) from first to \( n_m \)th are obtained as the references for Slave, where \( n_s \leq n_m \).

V. EXPERIMENTAL PRECISION STAGES

Fig. 9 shows the experimental precision stages applied in the synchronous control. The stage which consists of the AC-motor and the ball-screw in Fig. 9 (a) is called “Ball-screw stage”. The stage which consists of the linear-motor and the air-guide in Fig. 9 (b) is called “Nano-stage”.

Ball-screw stage is identified as Master. Nano-stage is identified as Slave. Frequency responses of the stages are shown in Fig. 10 and 11.

Each sampling period is
\[ T_{um} = T_{us} = T_{ym} = T_{ys} = T_u = T_y = 1 \text{ ms}. \]  
(20)

Each plant is
\[ P_m(s) = \frac{b_{pm0}}{s^2 + a_{pm1}s}, \]  
(21)  
\[ P_s(s) = \frac{b_{ps0}}{s^2 + a_{ps1}s}, \]  
(22)

Both Master and Slave are 2-order plants. Thus,
\[ n_m = 2, \ n_s = 2. \]  
(23)

Here, Each state of plants is represented as
\[ x_m = [y_m, \dot{y}_m]^T, \]  
(24)  
\[ x_s = [y_s, \dot{y}_s]^T. \]  
(25)

Each dead-time which occurs in the system is
\[ T_{dm} = T_y, \ T_{dom} = T_{dis} = T_{dos} = 0. \]  
(26)

VI. SIMULATIONS

Each control system is compared by simulations for the models of the experimental precision stages. Both feedback controllers \( C_{zm}[z] \) and \( C_{zs}[z] \) are constrained as PID controllers. The controller \( C_{zm}[z] \) of Master is designed by pole placement as the bandwidth is 30 Hz, and the controller \( C_{zs}[z] \) of Slave is designed by pole placement as the bandwidth is 60 Hz. Moreover, the observer gain \( H \) is designed as eigenvalues of \( (A_{zm}^{n_m} - A_{zm}^{n_m-1} H C_{dm}) \) are multiple roots of 100 Hz.

Target trajectories are shown in Fig. 12. The target position trajectory was generated by the 5th-order polynomial. The specification is shown in TABLE. I. \( T_{acc}, \ T_{con}, \ T_{dec} \) represent the acceleration time, the constant velocity time, the deceleration time and the target position in the trajectory. Each control system is evaluated by the position error between the real position \( y_m \) of Master and the real position \( y_s \) of Slave.

Fig. 13 shows the position error \( (y_m - y_s) \) as the plants are nominal. Perfect tracking is achieved in each control system except Real position FB.

Next, Fig. 14 and 15 show the position error \( (y_m - y_s) \) as the input disturbance 0.01 Nm occurs in Master. Comparing with Real position FB + PTC, the proposed system is better because the states of Master considering dead-time is obtained as references for Slave faster.
VII. EXPERIMENTS

Each control system is compared by experimental precision stages. The specification of target trajectories and controllers is similar to one of the simulation. Experimental results of position error \((y_m - y_s)\) for each control system are shown in Fig. 16.

A plant variation occurs in Master because of the non-linear friction characteristic caused by the ball-screw. Therefore, the position error occurs widely in Independent PTC not using the output of Master as the input of Slave.

For the same reason, the feedforward controller of Master does not work effectively. However, Real Position FB + PTC is a little bit better than Real Position FB in the acceleration and deceleration time.

On the other hand, the proposed system can achieve the precise synchronous positioning. The proposed system can be more resistant to the disturbance of master stage and plant variations because the state of Master considering with dead-time is obtained as references for Slave.

VIII. CONCLUSIONS

A synchronous position control system was proposed for a master-slave system required in industrial equipment. The synchronous position control system can be more resistant to the disturbance of master stage and plant variations by applying an observer, which compensates dead-times in the system. Simulations and experiment with experimental precision stages are performed to show the advantages of the proposed control system.

REFERENCES


APPENDIX

DERIVATION OF EQUATIONS (12) AND (13)

\[
\dot{x}_m[k - n_{dm} + 1] = A_{zm} x_m[k - n_{dm}] + b_{zm} u_m[k - n_{dm}] \\
+ H (y_m[k] - c_{zm} x_m[k - n_{dm}] - d_{zm} u_m[k - n_{dm}])
\]
\[ \hat{x}_m[k - n_{dm} + 2] = A_{zm} \hat{x}_m[k - n_{dm} + 1] + b_{zm} u_m[k - n_{dm} + 1] \]
\[ = A_{zm}^2 \hat{x}_m[k - n_{dm}] + [A_{zm} b_{zm}, b_{zm}] \begin{bmatrix} u_m[k - n_{dm}] \\ u_m[k - n_{dm} - 1] \end{bmatrix} \]
\[ + A_{zm} H (y_m[k] - c_{dm} \hat{x}_m[k - n_{dm}] - d_{dm} u_m[k - n_{dm}]) \]

In the same way,
\[ \hat{x}_m[k] = A_{zm} \hat{x}_m[k - 1] + b_{zm} u_m[k - 1] \]
\[ = A_{zm}^{n_{dm} - 1} \hat{x}_m[k - n_{dm}] + [A_{zm}^{n_{dm} - 1} b_{zm}, A_{zm}^{n_{dm} - 2} b_{zm}, \ldots, b_{zm}] \begin{bmatrix} u_m[k - n_{dm}] \\ \vdots \\ u_m[k - 1] \end{bmatrix} \]
\[ + A_{zm}^{n_{dm} - 1} H (y_m[k] - c_{dm} \hat{x}_m[k - n_{dm}] - d_{dm} u_m[k - n_{dm}]) \]

(12) is obtained. From this equation,
\[ e_m[k] = x_m[k] - \hat{x}_m[k] \]
\[ = A_{zm}^{n_{dm}} (x_m[k - n_{dm}] - \hat{x}_m[k - n_{dm}]) - A_{zm}^{n_{dm} - 1} H c_{dm} (x_m[k - n_{dm}] - \hat{x}_m[k - n_{dm}]) \]
\[ = (A_{zm}^{n_{dm}} - A_{zm}^{n_{dm} - 1} H c_{dm}) e_m[k - n_{dm}] \]

(13) is obtained.