Application of Perfect Tracking Control to Large-Scale High-Precision Stage

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Abstract: In the positioning system of the large-scale high-precision stage, the primary resonance mode appears in low frequency, though large-scale high-precision stages are used in industrial fields. In recent years, our research group has applied the perfect tracking control (PTC) method which can design the perfect inverse system of the plant to a large-scale high-precision stage to improve the control performance. Moreover, a synchronous position control based on PTC method is proposed as application to several stages. In early sections, the application of the PTC method to a large-scale stage is introduced. In latter sections, the application to synchronous position control is introduced.

Keywords: Precision stage, multirate control, fast and precise positioning, synchronous control.

1. INTRODUCTION

Large-scale high-precision stages are used in industrial fields such as manufacturing of semiconductors and liquid crystal panels (or displays). Fast and precise positioning control is very important technology related to the improvement of throughput and product quality. However, a large-scale stage has a low resonance mode because of its structure.

Therefore, the large-scale stage cannot depend on a high gain and high feedback control. A two degree of freedom control method that enables an accurate feedforward control should be applied.

Åström et al. (1984) indicate that the inverse system of the plant cannot be designed in discrete-time because unstable zero appears in the plant discretized by zero-order hold. Tomizuka (1987) proposed the zero phase error tracking controller (ZPETC) which compensates both phase and gain errors caused by un cancellable unstable zeros of discrete-time plant.

Whereas, Fujimoto et al. (2001) proposed a perfect tracking control (PTC) based on multi-rate feedforward control to eliminate the problem of unstable-zeros. PTC theoretically guarantees that a feedforward control based on an accurate inverse system for a nominal model can be realized and perfect tracking for every sampling period can be achieved as long as there is no unstable zero in the nominal model defined by a continuous-time system. Moreover, Fujimoto et al. (2006) also proposed vibration suppression PTC which deals with resonance modes of a plant, and applied to hard disk drives (HDDs).

In recent years, our research group has applied the PTC method to a large-scale high-precision stage to improve the control performance. This paper describes the application of the PTC method to a large-scale stage in early sections. From section 6, a synchronous position control based on PTC method is proposed as application to several stages. Industrial machines which have several stages often require synchronous positioning between these stages, thus the control is very important to decide final performance of the machines.

2. PERFECT TRACKING CONTROL

Fujimoto et al. (2001) proposed the perfect tracking control (PTC) which consists of the 2-DOF control system as shown in Fig. 1. This system has two samplers for the reference signal $r(t)$ and the output $y(t)$, and two holders for the input $u_o(t)$ and nominal output $y_o(t)$. Therefore, there exist sampling periods $T_r$, $T_y$ and $T_u$ which represent the periods of $r(t)$, $y(t)$, and $u(t)$, respectively. PTC applies the multirate feedforward control in which the control input $u(t)$ is changed $n$ times during one sampling period $T_r$ of reference input $r(t)$ as shown in Fig. 2.

Here, $n$ is the plant order. $H_M$ in Fig. 1 is the multirate holder which outputs the input $u[i] = [u_1[k] \cdots u_n[k]]^T$ (generated by the long sampling period $T_r$) on the short sampling period $T_u$. Here, the plant discrete time state space matrices $A$, $B$, $C$ and $D$ at the long sampling period $T_r$ can be derived as (2)
from the discrete time plant model at the short sampling period $T_u$ (1).

$$x[k + 1] = A_x x[k] + b_x u[k], \quad y[k] = c_x x[k] \quad (1)$$

$$[A|B] = \begin{bmatrix} A_x^n & A_x^{n-1} b_x & \cdots & A_x b_x & b_x \end{bmatrix}$$
$$[C|D] = \begin{bmatrix} c_x A_x & c_x b_x & \cdots & c_x b_x & 0 \end{bmatrix} \quad (2)$$

Since the matrix $B$ of (2) is non-singular in the case of controllable plant, PTC can be designed as

$$u_o[i] = B^{-1}(I - z^{-1}A)x_d[i + 1]$$
$$= \left[ \begin{array}{c} 0 \\ -B^{-1} A \\ I \\ \end{array} \right] x_d[i + 1] \quad (3)$$

$$y_o[i] = z^{-1} C x_d[i + 1] + D u_o[i]. \quad (4)$$

(3) is the stable inverse system of plant with input from the previewed desired trajectories. Therefore, perfect tracking is assured at the sampling period $T_r$.

The feedback control $C_2[z]$ suppresses the error between the output $y[k]$ and the nominal output $y_o[k]$ caused by disturbance or feedforward control imperfectness due to plant model error.

3. STRUCTURES AND MODELING OF STAGE

3.1 Structures of Stage

The large-scale high-precision stage with degrees of freedom XXYθ axes is the form with an individual multi-layer Z-axis air guide for each degree of freedom as shown in Fig. 3. It is called gantry stage.

3.2 Modeling of Gantry Stage

The gantry stage can be modeled as a mechanically separated SISO-system since it has an individual multi-layer Z-axis air guide for each degree of freedom. However, due to the multi-layer configuration, it is generally difficult to adjust the Z position of the actuator to match the Z position of the stage center of gravity. For this reason, if that structure is simplified, it can be illustrated as Fig. 4. At this time, if the $\theta$ is very small, the transfer function of the plant model is represented by (5). Here, this system is minimum-phase system.

$$P_2(s) = \frac{p_z}{f} = \frac{y}{u} = \frac{b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s} \quad (5)$$

Since PTC has the structure of a 2-DOF control system, the feedforward and feedback controller can be designed independently as explained in section 2. In this section, the design methods for each controller are explained.

4. DESIGN OF CONTROLLER

4.1 Design of Proposed Feedforward Controller

The controllable canonical form of the plant model is necessary for designing the feedforward controllers based on PTC.

In the gantry stage with (5), its equation is represented by

$$\dot{x}(t) = A_x x(t) + b_x u(t), \quad y(t) = c_x x(t). \quad (6)$$

$$x(t) = [z(t), z^{(1)}(t), z^{(2)}(t), z^{(3)}(t)]^T$$
where \( \dot{x}_n(t) \) is \( n \)th-order differentiation by definition of the controllable canonical form. Here, the equation of the plant model discretized by the short sampling period \( T_u \) is (1), the feedforward controllers based on PTC can be obtained from (3). The sampling period is \( T_u = T_y = T_r/4 \).

Moreover, the target trajectories of these stage controllers are different, too. In the gantry stage defined as rigid body and primary resonance modes, however, it cannot be given directly as a reference because \( y(t) \neq x_1(t) = z(t) \) and \( x_2(t) \) is a virtual position. Therefore, using the target trajectory filter of (7), it is inserted to generate the target trajectory \( z_d(t) \) from the real position trajectory \( y(t) \). Then, the zeros of the plant model are canceled, and perfect tracking control can be realized.

\[
R(s) = \frac{b_0}{b_2 s^2 + b_1 s + b_0}
\]

(7)

4.2 Design of Feedback Controller

The P-PI control system which has a velocity loop inside a position loop is used as the feedback controller. In the case of using an approximated simple plant model, feedback controllers are apt to be designed conservatively. Therefore, those parameters were designed through a trial and error process in order to avoid a conservative design.

4.3 Design of Conventional Feedforward Controller

In the experiments in the following section, the proposed method is compared with a conventional method. The conventional feedforward controller used in the comparison will be explained below.

A conventional feedforward controller assumes that the plant model is equal to a rigid model. And, using the differential trajectory of the target trajectory \( r(t) \), the output of controller \( u_{FF}(t) \) is calculated as

\[
u_{FF}(t) = h_2 \ddot{r}(t) + h_3 \dot{r}(t),
\]

where the coefficients of \( h_2 \) and \( h_3 \) are theoretically equal to the mass and viscosity of a rigid model. However, in reality, they are determined through a trial and error process. As shown in Fig. 6, this controller is combined with previous feedback controller. As a result, the conventional control system is single rate control.

5. EXPERIMENT OF POSITIONING BY PTC

5.1 Definition of Specification and Control Performance

Each sampling period of the controller is \( T_y = T_u = 1/3 \) ms, which is the same with the actual controller.

Fig. 6. Conventional control system.

Fig. 7. Target trajectory 1.

Fig. 8. Frequency responses of position loop.
Since the state variables (position, velocity, · · · ) exclude dead-time of the system as shown in Fig. 11. Moreover, we proposed the system with an observer configuration. However, the system cannot consist of only the feedback controller has been proposed by Nakamura et al. (2004). However, the system cannot achieve the sampling period in Slave, the system can be more resistant to the disturbance of master stage and plant variations.

7. CONTROL METHOD OF PROPOSED SYNCHRONOUS POSITIONING

The class of the problem is assumed as $T_{um} = T_{us} = T_{pm} = T_{gs} = T_{u} = T_{y}$ easily though many various cases are considered about sampling periods of Master and Slave. Dead-time which occurs in this work and Slave are discussed.

Dead-time which occurs in the plants are defined as $T_{dim}$ and $T_{dis}$. Dead-time which occurs in the sensors are defined as $T_{dom}$ and $T_{dos}$ as a delay exists in the output. Fig. 12 shows the plant with dead-time. Here, the state $x_m(kT_y + T_{dim})$ of Master can be estimated at a time $kT_y$ because the dead-time of Master is summed up in the output of Master as shown in Fig. 12. Since the state $x_m(kT_y + n_sT_u + T_{dis})$ of Master is needed to synchronize the real position $y_s(t)$ of Slave and the real position $y_m(t)$ of Master, the estimated state $x_m(kT_y + T_{dim})$ has to be stepped for $n_sT_u + T_{dis} - T_{dim}$ periods moreover.

7.1 Definition of State Equations of Plants

The continuous time state equation of the plant of Master except dead-time is represented as

$$\begin{align*}
\dot{x}_{pm}(t) &= A_{c_{pm}}x_{pm}(t) + b_{c_{pm}}(u_m(t) - d_m(t)) \\
y_m(t) &= c_{c_{pm}}x_{pm}(t)
\end{align*}$$

(10)
as a controllable canonical form. The continuous time state equation of the disturbance of Master is represented as

$$\begin{align*}
\dot{x}_{dm}(t) &= A_{c_{dm}}x_{dm}(t) \\
d_m(t) &= c_{c_{dm}}x_{dm}(t)
\end{align*}$$

(11)
The augmented continuous time state equation of Master can be represented as

\[
\begin{align*}
\dot{x}_m(t) &= A_{cm}x_m(t) + b_{cm}u_m(t), \\
y_m(t) &= c_{cm}x_m(t),
\end{align*}
\]

(12)

where \( x_m = [x_{pm}, x_{dm}]^T \). Then, the augmented discrete time state equation of (12) per \( T_u \) is represented as

\[
\begin{align*}
x_m[k+1] &= A_{zm}x_m[k] + b_{zm}u_m[k], \\
y_m[k] &= c_{zm}x_m[k].
\end{align*}
\]

(14)

In Slave, these equations are defined similarly as the subscipt \( m \) is changed into \( s \).

### 7.2 Estimation of States of Master

The dead-time of Master summed up in the output of Master is defined as

\[ T_{dm} = T_{dim} + T_{dom}. \]

(15)

Thus, the output of Master delays for the dead-time \( T_{dm} \) as Fig. 13.

Here, the state \( x_m(kT_y + T_{dim}) \) of Master at the time \( kT_y \) is estimated as the state \( \hat{x}_m[k] \) by an observer.

In the case of \( (n_{dm} - 1)T_y < T_{dm} \leq n_{dm}T_y \) \( (n_{dm} = 1, 2, \ldots) \) at the time \( kT_u \), the available output signal \( y_m[k] \) is represented as

\[
y_m[k] = c_{dm}x_m[k - n_{dm}] + d_{dm}u_m[k - n_{dm}],
\]

(16)

\[
c_{dm} = c_{cm}e^{\int c_{zm}T_{dm} - T_{T_y}},
\]

(17)

\[
d_{dm} = \int_0^{T_{dm}} c_{zm}e^{\int c_{zm}T_{dm} - T_{T_y}}d\tau.
\]

(18)

The state \( x_m[k] \) of the plant of Master can be estimated by the equation as follows:

\[
\hat{x}_m[k] = A_{zd_m}^n\hat{x}_m[k - n_{dm}]
\]

\[
+\left[A_{zd_m}^{n_{dm}-1}b_{zd_m}A_{zd_m}^{n_{dm}-2}b_{zd_m} \cdots b_{zd_m} \right] \tilde{u}_m[k - n_{dm}]
\]

\[
+ A_{zd_m}^{n_{dm}-1}H(ym[k] - c_{zm}\hat{x}_m[k - n_{dm}] - d_{zm}u_m[k - n_{dm}])(19)
\]

\[
e_m[k] = (A_{zd_m}^n - A_{zd_m}^{n_{dm}-1}H)c_{zm}e_m[k - n_{dm}].
\]

(20)

### 7.3 Generation of References for Slave

Since the state \( \hat{x}_m[k] \) of Master at \( kT_y \) has been estimated, the state \( \hat{x}_m[i + N + \sigma] \) which is stepped \( n_sT_y + T_{dis} - T_{dim} \) periods ahead is to be estimated per \( n_s \) periods. Here, \( [i] = (i \cdot n_sT_y), N = 0, 1, \ldots \) and \( 0 \leq \sigma < 1 \).

In the case that \( n_sT_y + T_{dis} - T_{dim} < 0, \hat{x}_m[k] \) has only to be delayed for this period.

In the case that \( n_sT_y + T_{dis} - T_{dim} \geq 0 \), the state \( \hat{x}_m[i + N + \sigma] \) is estimated as

\[
\hat{x}_m[i + n_s + \sigma] = A_{dm}\hat{x}_m[i] + B_{dm}u_m[i],
\]

(21)

\[
A_{dm} = e^{A_{cm}(N+\sigma)T_y},
\]

(22)

\[
B_{dm} = [b_{dm1}, \ldots, b_{dmN}, b_{dm(N+1)}],
\]

(23)

\[
u_m[i] = [u_{m1}, \ldots, u_{mN}, u_{m[N+1]}]^T[i],
\]

(24)

where,

\[
\begin{align*}
b_{dmj} &= A_{zm}^{j-1}b_{zm}(j \leq N) \\
b_{dm(j+1)} &= \int_0^{T_{dm}} e^{A_{cm}T_{dm}}b_{zm}d\tau, \\
\{u_{m1} &= u_m[k] \\
u_{mj} &= u_m[k+j-1](2 \leq j \leq N + 1)
\end{align*}
\]

(25)

(26)

Here, \( u_{omat} \) represents a nominal input calculated by the multirate feedforward controller previously.

Thus, the elements of the state \( \hat{x}_m[i + N + \sigma] \) from first to \( n_{om} \)th are obtained as the references for Slave, where \( n_s \leq n_{om} \).

### 8. EXPERIMENT OF SYNCHRONOUS SYSTEM

#### 8.1 Experimental Precision Stages

The experimental precision stages applied in the synchronous control are prepared. The stage which consists of the AC-motor and the ball-screw is identified as Master. The stage which consists of the linear-motor and the air-guide is identified as Slave. Frequency responses of the stages are shown in Fig. 14 (a) and (b).

Each sampling period is

\[ T_{um} = T_{us} = T_{ym} = T_{ys} = T_u = T_y = 1 \text{ ms}. \]

(27)

Each plant is

\[
P_m(s) = \frac{b_{pm0}}{s^2 + a_{pm1}s}, \quad P_s(s) = \frac{b_{ps0}}{s^2 + a_{ps1}s}.
\]

(28)

Both Master and Slave are 2-order plants. Thus, \( n_m = 2, n_s = 2 \).

Here, each state of plants is represented as

\[ x_m = [y_m, \dot{y}_m]^T, \quad x_s = [y_s, \dot{y}_s]^T. \]

(30)

Each dead-time which occurs in the system is

\[ T_{dim} = T_y, \quad T_{dom} = T_{dis} = T_{dos} = 0. \]

(31)
8.2 Definition of Specification and Control Performance

Both feedback controllers $C_{2m}[z]$ and $C_{2s}[z]$ are constrained as PID controllers. The controller $C_{2m}[z]$ of Master is designed by pole placement as the bandwidth is 30 Hz, and the controller $C_{2s}[z]$ of Slave is designed by pole placement as the bandwidth is 60 Hz. Moreover, the observer gain $H$ is designed as eigenvalues of $(A^n_{2m} - A^n_{2m-1} H c_{dm})$ are multiple roots of 100 Hz.

Target trajectories are shown in Fig. 15. The target position trajectory was generated by the 5th-order polynomial. The specification is shown in Table 2. $T_{acc}$, $T_{con}$, $T_{dec}$ and $A^{ref}$ represent the acceleration time, the constant velocity time, the deceleration time and the target position in the trajectory. Each control system is evaluated by the position error between the real position $y_m$ of Master and the real position $y_s$ of Slave.

8.3 Experimental results

Each control system is compared by experimental precision stages. Experimental results of position error $(y_m - y_s)$ for each control system are shown in Fig. 16. The standard deviation $3\sigma$ of each position error is shown in Table 3.

A plant variation occurs in Master because of the non-linear friction characteristic caused by the ball-screw. The proposed system can achieve the precise synchronous positioning. The proposed system can be more resistant to the disturbance of master stage and plant variations because the state of Master considering with dead-time is obtained as references for Slave.

9. CONCLUSION

PTC method was applied to fast and precise positioning of the large-scale high-precision stage which has low resonance mode. The results of this paper prove that the PTC method is a very effective strategy for achieving high-speed and high-precision tracking control.

Moreover, a synchronous position control system was proposed for a master-slave system required in industrial equipment. The synchronous position control system can be more resistant to the disturbance of master stage and plant variations by applying an observer, which compensates dead-time in the system. Experiments with experimental precision stages are performed to show the advantages of the proposed control system.

**REFERENCES**


