Torque Ripple Suppression Control for PM Motor with High Bandwidth Torque Meter

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Abstract—PM motor drive systems are widely used in many industrial applications. However, PM motors have the fluctuation of the magnetic field distribution which produces the torque ripple. Dead time of the inverter, offset of sensors and current measurement errors lead to the torque ripple, too. In this paper, we proposed a torque ripple suppression method with PWM-hold model, in which the torque ripple is measured by a high-bandwidth torque meter. To suppress the torque ripple, we propose a method to generate the compensation signal by using the information from the torque meter. Finally, we show the advantages of the proposed method by simulations and experiments with a SPMSM.

Index Terms—PMSM, high bandwidth torque meter, perfect tracking control, suppression of torque ripple

I. INTRODUCTION

Recently, permanent magnet (PM) motor drive systems are widely used for industrial drives and automotive applications. However, in case of surface permanent magnet synchronous motor (SPMSM), an imperfect sinusoidal flux distribution causes torque ripple. In case of interior permanent magnet synchronous motor (IPMSM), the motor structure and a placement of permanent magnet generates torque ripple. Moreover, the dead time of the inverter, the offset of sensors and current measurement error causes the torque ripple. The torque ripple leads vibration and noises. Thus, it is necessary to suppress the torque ripple.

Many researches and developments have been done on harmonic current and torque ripple suppression. In [1], harmonic current suppression method has proposed in rotating coordinate which is synchronized with the harmonic component for IPMSM. In addition, torque ripple suppression method was developed with plant model including harmonic current for the IPMSM which has distorted induced voltage in [2]. Moreover, torque ripple suppression control method which is modeling torque ripple using motor position with feedforward compensation has proposed in [3]. In [4], cogging torque compensation method of the linear motor with adaptive robust control has proposed. Furthermore, torque ripple suppression method of brushless DC motor with direct torque control has proposed in [5]. In [6], the practical design considerations of a low torque ripple PMSM drive for electric power steering (EPS) application has proposed. In [7], the torque ripple minimization method with an adaptive feedback structure has proposed.

Our research group proposed torque ripple suppression control method with high resolution encoder in [8]. In this paper, a torque ripple suppression method is developed, in which the torque ripple is measured by a high-bandwidth torque meter. Generally, torque meter is unsuitable for detecting and controlling torque ripple because of its low bandwidth. However, high-bandwidth torque meter which has 5kHz cutoff frequency was developed recently.

In this paper, a novel method to detect and suppress the torque ripple of three phase SPMSM is proposed. We focus that the torque ripple changes periodically depending on the position of rotor and apply the current control based on perfect tracking control [9]. Finally, we show the advantages of the proposed method by simulations and experiments.
IV. CONTROL SYSTEM DESIGN

A. Discretization of Plant

In general, the discrete-time model of the controlled plant is obtained with the zero-order-hold. The zero-order-hold discretization approximately assumes that the inverter can output the arbitrary voltage. In the case of controlling instantaneous values precisely, it is unsuitable. The single-phase inverter output can take only 0 or ±E. The switches are turned on and off once during each interval \( T_u \) such that a voltage pulse of magnitude \( E \) (or \(-E\)) and width \( \Delta T \) centered in the interval \( T_u \). Then, this paper uses PWM hold model which can evaluate the instantaneous voltage precisely by regarding pulse width as control input. According to [11], the plant of the inverter drive system can be discretized based on the PWM hold, the discrete time model of the plant is derived by

\[
x[k + 1] = A_s x[k] + B_s \Delta T[k], y[k] = C_s x[k]
\]

\[
A_s = e^{A_s T_u}, B_s = e^{A_s T_u/2} B_s, C_s = C_e.
\]

B. Input Generation of Three-phase Inverter

The control input of the proposed method is the switching times \( \Delta T_d \) and \( \Delta T_q \) because the control system is designed by the dq-model. In order to apply the PWM hold model to PM motor, the input for PWM pulse of the three-phase system has to be generated.

The triangle-wave PWM is considered. The input voltage \( V_{dc} \) of the three-phase inverter is defined. The discrete model of (6) for the dq-system is designed as \( E = V_{dc}/3 \) in (6) (in the case of absolute transforms \( E = V_{dc}/2 \)). \( \Delta T_d \) and \( \Delta T_q \) are transformed into \( \Delta T_u \) and \( \Delta T_v \) and \( \Delta T_w \) by eq(3)/phase relative transform of

\[
\begin{bmatrix}
\Delta T_u \\
\Delta T_v \\
\Delta T_w
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\cos(\theta - \pi/2) & -\sin(\theta - \pi/2) \\
\cos(\theta - 2\pi/3) & -\sin(\theta - 2\pi/3)
\end{bmatrix}
\begin{bmatrix}
\Delta T_d \\
\Delta T_q
\end{bmatrix}.
\]

Then, the inputs \( u, v \) and \( w \) of the sine-triangle PWM are given by

\[
u[k] = \frac{\Delta T_u[k]}{T_u}, v[k] = \frac{\Delta T_v[k]}{T_v}, w[k] = \frac{\Delta T_w[k]}{T_w}.
\]

C. Perfect Tracking Control (PTC) [13]

PTC consists of the 2-degree-of-freedom control system as shown in Fig.4. The feedforward controller is a stable inverse system of the plant, and assures the perfect tracking for a nominal plant at the sample point. Because the plant is the first order system, PTC is achieved by a usual singlerate control as shown in Fig.4. The design of the feedforward controller is described. By discretizing (5) with PWM hold model, (9) is obtained.

\[
x[k + 1] = A x[k] + B u[k], y[k] = C x[k]
\]

\[
A = e^{-\frac{A_s T_u}{2}}, B = e^{-\frac{A_s T_u}{4}} \frac{1}{L} V_{dc}, C = 1.
\]

Therefore, a stable inverse system of the plant is obtained as (9), and a nominal output is given as (10),

\[
u_0[k] = B^{-1}(1 - z^{-1} A)x_d[k + 1],
\]
The first term of (11) is reluctance torque. The second term is the flux linkage excited by \( i_s \), and the third term is cogging torque. Now, we define \( d\Psi_{sr}(\theta)/d\theta \) as \( \Psi'_{r}(\theta) \) and suppose that current of winding and flux have fundamental components and harmonic components each in. Moreover, torque components generated by the fundamental component \( \Psi'_o \) and the fundamental current \( i_o \) is defined as \( T_{Mo} \). The torque component generated by harmonic current \( i_h \), harmonic flux \( \Psi'_h \) and cogging torque is also defined as \( T_{cog}(\theta) \).

\[
T_M(t) = T_{Mo}(t) + T_r(\theta,i),
\]

\[
T_{Mo}(t) = i_o(t)\Psi'_o
\]

\[
T_r(\theta,i) = i_h(t)\Psi'_h(\theta) + i_o(t)\Psi'_o(\theta) + i_h(t)\Psi'_h(\theta) + T_{cog}(\theta),
\]

where \( T_{cog}(\theta) \) is the cogging torque and suffix \( o \) refers fundamental components, suffix \( h \) refers harmonic components. Therefore, the motion equation of rotation system about motor including torque ripple is represented as

\[
J_M\ddot{\theta}(t) = K_i(i(t) + B_M\omega(t) - T_s(t))
\]

\[
K_i(t) = \Psi'_o(i_o(t) + i_h(t)),
\]

where \( T_s \) is the torque of a shaft torsion and suppose that this torque can be detected from torque meter. \( i_c(t) \) is the current additionally generated by PTC after the switch 2 turns on. When the switch 1 turns on, i.e., \( i_c(t) = 0 \), torque ripple generated in motor can be estimated by

\[
\hat{T}_r(\theta,i) = T_s(t) - K_i(i(t) + B_M\omega(t) + J_M\ddot{\theta}).
\]

\[
\hat{T}_r(\theta,i) \text{ calculated by (14) is stored in the in memory as data of periodic disturbance. The current reference which suppresses } \hat{T}_r(\theta,i) \text{ by } K_i \text{ and desired torque } T^*_r(t) \text{ is generated as (15).}
\]

\[
i^*(t) = i_{ref}(t) - i'_{ref}(t)
\]

\[
i_{ref}(t) = \frac{1}{K_t} T^*_r(t), i'_{ref}(t) = \frac{1}{K_t} \hat{T}_r(\theta,i).
\]

In this paper, \( \hat{T}_r(\theta,i) \) is assumed to be independent of current \( i \) because the current reference is set to constant. So we consider only the dependence on position \( \theta \).

\section{SIMULATION}

The torque ripple suppression simulation was carried out to confirm the effectiveness of the proposed method. In this simulation, the torque ripple includes 240 [Hz] component and 560 [Hz] component. These torque ripple components are detected by the experiment. The motor speed is controlled by the load motor at 800 [rpm]. The torque ripple compensation begins from 2.0 [s]. The results of simulation are shown in Fig.6. Fig.6(a)–(c) show that torque ripple becomes smaller by the proposed compensation. Fig.6(d) shows the waveform of the control input \( u_0 \) and its spectrum is shown in Fig.6(e). Here, \( q \) axis current is shown in Fig.6(f). Moreover, its spectrum before compensation is shown in Fig.6(g) where the dc component 30 [A] is not displayed. Fig.6(h) shows current spectrum after compensation. As shown in Fig.6(b), 240 [Hz] and 560 [Hz] current are injected to cancel the torque ripple.
Fig. 6. Simulation results of torque ripple suppression.
TABLE I
PARAMETERS OF PLANT

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductance ( L )</td>
<td>500 ( \mu \text{H} )</td>
</tr>
<tr>
<td>Resistance ( R )</td>
<td>33 ( \text{m}\Omega )</td>
</tr>
<tr>
<td>Inertia (motor) ( J_M )</td>
<td>( 2.14 \times 10^{-3} ) ( \text{kgm}^2 )</td>
</tr>
<tr>
<td>Viscous friction (motor) ( B_M )</td>
<td>( 1.23 \times 10^{-4} ) ( \text{kg}/(\text{m}\cdot\text{s}) )</td>
</tr>
<tr>
<td>Induced voltage constant ( K_c )</td>
<td>( 1.59 \times 10^{-2} ) ( \text{V}/\text{s}/\text{rad} )</td>
</tr>
<tr>
<td>Torque constant ( K_t )</td>
<td>( 1.11 \times 10^{-1} ) ( \text{N}\cdot\text{m}/\text{A} )</td>
</tr>
<tr>
<td>Pairs of Poles (motor)</td>
<td>7</td>
</tr>
<tr>
<td>Inertia (load) ( J_L )</td>
<td>( 1.63 \times 10^{-4} ) ( \text{kgm}^2 )</td>
</tr>
<tr>
<td>Viscous friction (load) ( B_L )</td>
<td>( 3.26 \times 10^{-6} ) ( \text{kg}/(\text{m}\cdot\text{s}) )</td>
</tr>
<tr>
<td>Pairs of Poles (load)</td>
<td>3</td>
</tr>
<tr>
<td>Spring constant ( K_s )</td>
<td>( 1.45 \times 10^1 ) ( \text{Nm}/\text{rad} )</td>
</tr>
<tr>
<td>Encoder resolution</td>
<td>24,000 pulse/rev</td>
</tr>
</tbody>
</table>

VI. EXPERIMENT

The torque ripple suppression experiment was carried out by using SPMSM to confirm the effectiveness of the proposed method. The specification of the SPMSM which is used in the experiment is shown in Table I. The load motor is driven by the speed control mode and controls the motor speed at 800 [rpm]. The test motor is driven by the torque control mode. The torque ripple compensation starts from 0.0 [s]. When compensation signal is generated in the proposed method, the averaging is used during 100 \( T_d \) to avoid the non-periodic components. In this case, \( T_d \) is 75 [ms]. Moreover, to remove the noise caused by the current sensor, Q filter is used at the output of PSG in Fig.5. The Q filter is given as

\[
r_f[k] = \frac{z + \gamma + z^{-1}}{\gamma + 2} r[k]
\]

where \( r[k] \) is output of PSG, \( r_f[k] \) is output of Q filter. This filter is a low-pass filter with zero phase-delay, and (16) needs the one-sample ahead value, which is available from the memory as \( r[k+1] = r[k-N_d+1] \). Moreover, the smaller \( \gamma \geq 2 \) has bigger roll-off while the disturbance rejection performance becomes poorer. In the experiment, \( \gamma \) is 2 and cut-off frequency becomes 1.8 [kHz]. Moreover, notch filter is used to suppress 480 [Hz] mechanical resonance mode. In the case of the simulation, \( T_{ev} \) in Fig.3 can be measured directly. However, in the experiment, it is impossible to measure \( T_{ev} \) directly. We evaluate experimental results by using (17).

\[
T_{ev} = J_M \omega_M + B_M \omega_M
\]

where \( B_M \) and \( J_M \) is the nominal value. The results of experiment are shown in Fig.7. Fig.7(a) shows torque spectrum before compensation and (b) shows after compensation. They prove that the torque ripple is suppressed after the compensation. Fig.7(c) shows the waveform of the control input \( u_0 \) and its spectrum is shown in Fig.7(d). Here, \( q \) axis current before compensation is shown in Fig.7(e) and after compensation is shown in Fig.7(f). Moreover, those spectrum is shown in Fig.7(g), (h). In Fig.7(a), 240 [Hz] component is sixth-order torque ripple of the load motor, and 560 [Hz] component is sixth-order torque ripple of the test motor. As shown in Fig.7(b), 240 [Hz] component is suppressed to 38.01% after compensation and 560 [Hz] component is also suppressed to 36.7% after compensation. As shown in Fig.7(d), 240 [Hz] component and 560 [Hz] component are larger than any other components. Therefore, as shown in Fig.7(h), the current which suppresses the torque ripple is generated.

VII. CONCLUSION

In this paper, the torque ripple suppression control method with high-bandwidth torque meter was discussed. It is proved that the torque ripple can be detected with the torque meter and compensation signal can be generated with that information.

To prove the effectiveness of the proposed method, simulations and the experiments of the torque ripple suppression control were performed with the SPMSM. It was shown that proposed method has excellent performance.

REFERENCES

Fig. 7. Experimental results of torque ripple suppression.