Proposal of Multirate PWM Positioning Control Considering Resonance Mode for Precision Stage

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Abstract—Motion control techniques are employed on nanoscale positioning in industrial equipments, for example, NC machine tools, exposure systems, and so on. The advanced motion control techniques are based on the precise current control. However, speed-up of the precise current response has a serious limitation because of the carrier period of an inverter. In addition, the positional response has to be slower than the current response. In our past paper, we designed and fabricated an experimental precision stage. Then, we achieved a novel ultrahigh-speed nanoscale positioning based on multirate PWM control. The positional error was in 100 nm. The positioning time was 2 ms which was only 20 times as long as the carrier period. However, it was difficult to achieve faster and preciser positioning because of resonance modes of the stage. In this paper, we propose multirate PWM control considering resonance mode. Finally, simulations and experiments are performed to show the advantages of the proposed method.

I. INTRODUCTION

Digital control of servomotors has been researched actively thanks to high-speed switching of PWM inverter and improved arithmetic processing of digital signal processor (DSP). Nowadays, motion control techniques are employed on nanoscale positioning in precision mechanical equipment, for example, NC machine tools, exposure systems, and so on. Especially, the high-precision motion control is required to achieve nanoscale positioning for stages of exposure systems. There are many researches of high precision positioning of the stage [1], [2] and [3].

Then, the high-precision motion control is founded on precise current control of the motor. The performance of the current control is very important. However, speed-up of the precise current response has a serious limitation because of the carrier period of an inverter. In addition, the positional response has to be slower than the current response. There is a research that field programmable gate array (FPGA) is developed for motor drives to solve the problem of the limitation of the carrier period [4]. In the other way of approach with control technology, there are advantages of reduction of cost and switching loss.

The authors applied multirate PWM control for positioning of a servo motor. Positioning in 20 ms was achieved by using DSP in the case that carrier period was 2 ms [5]. Moreover, an experimental precision stage was designed and fabricated [6]. Then, a novel ultrahigh-speed nanoscale positioning was achieved based on multirate PWM control. The positional error was in 100 nm. The positioning time was 2 ms which was only 20 times as long as the carrier period.

However, it was difficult to achieve faster and preciser positioning because of resonance modes of the stage. In this paper, we propose multirate PWM control considering resonance mode for more precise positioning. Finally, simulations and experiments are performed to show the advantages of the proposed method.

II. NANO-STAGE

An experimental high-precision stage was designed and fabricated to research ultrahigh-speed nanoscale positioning. The experimental stage is called “Nano-stage” below.

A. Constitution of Nano-stage

Fig. 1 shows the overview of Nano-stage. Nano-stage driven by linear motor is the stage whose friction is almost zero because of using the air guide. Moreover, Nano-stage has two linear encoders to measure both the motor part which is the drive and the stage part which is the load. The resolution of the linear encoders is 1 nm/pulse respectively to achieve nanoscale positioning.

Nano-stage can be switched to two modes of the rigid mode and the two-inertia mode. In the two-inertia mode, the motor and the stage parts are connected by leaf springs. The resonance characteristic can be changing by replacing the leaf springs. Then, Nano-stage can be switched to the rigid mode because the motor and the stage parts are fixed by plates. In this paper, Nano-stage is treated as the rigid mode. Then, calculating the barycentric position of the stage with upper and lower encoders, the position is assumed as the real position of the stage.

B. Characteristics of Nano-stage

Pole-zero canceling proportional-integral current controller was designed for the current loop to be first-order system whose band frequency was 1 kHz. Table. I shows parameters of Nano-stage. Fig. 2 shows the frequency response from the current reference to the velocity of the stage. Here, the solid
TABLE I
PARAMETERS OF NANOSTAGE.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductance $L$</td>
<td>6.4 mH</td>
</tr>
<tr>
<td>Resistance $R$</td>
<td>13.1 Ω</td>
</tr>
<tr>
<td>Mass $M$</td>
<td>14.3 kg</td>
</tr>
<tr>
<td>Viscosity $B$</td>
<td>22.8 N/(m/s)</td>
</tr>
<tr>
<td>Thrust coefficient $K_f$</td>
<td>28.5 N/A</td>
</tr>
<tr>
<td>Back-emf constant $K_v$</td>
<td>9.3 V/(m/s)</td>
</tr>
<tr>
<td>Size</td>
<td>2.27 × 21 cm²</td>
</tr>
</tbody>
</table>

A continuous-time state equation of a plant is represented by

$$
\dot{y}(t) = \frac{1}{\tau s + 1} \cdot \frac{1}{Ms + B} \cdot G_{ vib }(s),
$$

(1)

where $\tau = 1/(2\pi 1000)$. Note that there is the resonance mode in spite of the stage of the rigid mode. The resonance mode is considered in Section IV.

III. MULTIRATE PWM POSITION CONTROL

Multirate PWM control system is a kind of perfect tracking control (PTC) system [7] which is designed for a plant model discretized based on PWM hold. Multirate PWM control is designed considering a current loop and instantaneous values of PWM pulse precisely. High-speed positioning in several carrier sampling periods which are shorter than the response of the current loop can be achieved. In output voltage control of a single-phase inverter, a high performance control which was tracking the output voltage on an arbitrary waveform was achieved [8].

A. Discrete Model Based on PWM-hold

In order to discretize a plant model, a zero-order hold is applied generally. However, in the case that a single-phase inverter (or a four-quadrant chopper) of Fig. 3 actuates DC-motor, the inverter can output not arbitrary output vector $V[k]$ but only 0 or ±$E[V]$ as Fig. 4. Therefore, in order to control instantaneous values precisely, the zero-order hold is unsuitable because the precise discrete model is based on the PWM hold of Fig. 4. The plant model of a motor actuated by an inverter can be discretized based on the PWM hold as follows [9].

A continuous-time state equation of a plant is represented by

$$
\begin{align*}
\dot{x}(t) &= A_s x(t) + b_s u(t) \\
y(t) &= c_s x(t)
\end{align*}
$$

(2)

The precise discrete model in which the input $u[k]$ is the switching time $ΔT[k]$ is obtained as

$$
\begin{align*}
x[k + 1] &= A_s x[k] + b_s ΔT[k] \\
y[k] &= c_s x[k]
\end{align*}
$$

(3)

$$
A_s = e^{A_s T_s}, \quad b_s = e^{A_s T_s/2} b_e E, \quad c_s = c_c
$$

(4)

When $ΔT[k]$ is negative, $-E[V]$ is outputted. (3) can be applied for AC-servo system driven by 3-phase inverter with vector control.

B. Perfect Tracking Control

Perfect tracking control (PTC) which consists of the 2-DOF control system as shown in Fig. 5. This system has two samplers for the reference signal $r(t)$ and the output $y(t)$, and one holder for the input $u(t)$. Therefore, there exist sampling periods $T_r$, $T_u$, and $T_e$ which represent the periods of $r(t)$, $y(t)$, and $u(t)$, respectively. PTC applies the multirate feedforward control in which the control input $u(t)$ is changed $n$ times during one sampling period $T_e$ of reference input $r(t)$ as shown in Fig. 6. Here, $n$ is the plant order. $H_M$ in Fig. 5 is the multirate holder which outputs the input $u[i] = [u_1[k], \ldots, u_n[k]]^T$ (generated by the long sampling period $T_e$) on the short sampling period $T_u$.

Here, the matrices $A$, $B$, $C$, and $D$ are given as

$$
\begin{align*}
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} =
\begin{bmatrix}
A_s^n & A_s^{n-1} b_s & \cdots & A_s b_s & b_s \\
c_s & c_s A_s & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
c_s A_s^{n-2} & c_s A_s^{n-3} b_s & \cdots & c_s b_s & 0
\end{bmatrix}
\end{align*}
$$

(5)
The plant of ‘Conventional 2’ is described by

\[ y_{0}[i] = z^{-1}C_{x}x_{d}[i + 1] + Du_{0}[i]. \]

(6) is the stable inverse system of the plant as the references are state variables \( x_{d}[k + 1] \). Therefore, the perfect tracking can be assured on the sampling period \( T_{s}. \)

Moreover, the feedback control \( C_{2}[z] \) suppresses the error between the output \( y_{0}[k] \) and the nominal output \( y_{0}[k] \) to assure robustness only when disturbances or plant variations exist.

### C. Original PTC system

PTC is designed for the third-order plant as the current loop is the ideal first-order system. This method is named ‘Conventional 2’. The design method for the second-order plant ignored current loop is named ‘Conventional 1’.

The plant of ‘Conventional 1’ is described by

\[ y = \frac{1}{Ms^2 + Bs}. \]

The plant of ‘Conventional 2’ is described by

\[ y = \frac{1}{\tau s + 1} \cdot \frac{1}{Ms^2 + Bs}. \]

(9) These plants are discretized by zero-order hold, then PTC is designed.

### D. Constitution of Multirate PWM Position Control System

In the rigid mode, \( q \) axis model of Nano-stage is shown in Fig. 7. Therefore, a control system can be designed for the model with vector control. The transfer function from \( V_{inv} \) to \( y \) is described by

\[ y = \frac{K_{1}}{V_{inv}} \cdot \frac{MLs^3 + (MR + LB)s^2 + (BR + K_{c}K_{q})s}{MLs^3 + (MR + LB)s^2 + (BR + K_{c}K_{q})s}. \]

The controllable canonical form of (10) is given by

\[ 
\begin{bmatrix}
A_{c} & b_{c} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
M & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
K_{1} \\
0
\end{bmatrix},
\]

(11) where \( x = [y \ y \ y]^{T}. \)

Also, the transfer function from \( V_{inv} \) to \( i \) is described by

\[ i_{c} = \frac{MS^2 + Bs}{V_{inv}} \cdot \frac{MLs^3 + (MR + LB)s^2 + (BR + K_{c}K_{q})s}{MLs^3 + (MR + LB)s^2 + (BR + K_{c}K_{q})s}. \]

The output equation of (13) is represented by

\[ y = c_{e}^{T}x, \quad c_{e}^{T} = \left[ \begin{array}{c}
0 \\
\frac{b_{c}}{K_{1}} \\
\frac{b_{c}}{K_{1}} \\
\frac{b_{c}}{K_{1}}
\end{array} \right]. \]

(14) The multirate PWM position control system considering PWM hold can be designed as Fig. 8. The current controller \( C_{PI}[z] \) and the position controller \( C_{2}[z] \) operate only when the errors between the nominal outputs and the actual outputs exist.

### E. Input Generation of Three-phase Inverter

In the case that the plant is AC motor like a linear motor, the input is the switching times \( \Delta T_{d} \) and \( \Delta T_{q} \) because the control system is designed by the \( dq \)-model. However, in order to apply the three-phase inverter, the input for PWM pulse of the three-phase system has to be generated. The generation method is explained. Here, coordinate transform matrices are represented as absolute transforms.

The input voltage of the three-phase inverter is defined as \( V_{dc}. \) The discrete model of (3) for the \( dq \) model is designed.
as $E = V_{dc}$ in (4). $\Delta T_d$ and $\Delta T_q$ are transformed into $T_{\alpha}$ and $T_{\beta}$ by the $dq/2$-phase transform as follows:

$$
\begin{bmatrix}
T_{\alpha} \\
T_{\beta}
\end{bmatrix}
= \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\Delta T_d \\
\Delta T_q
\end{bmatrix}.
$$

(15)

Then, $\Delta T_{\alpha}$ and $\Delta T_{\beta}$ are transformed into $T_{uv}$, $T_{uw}$, and $T_{ww}$ by the 2-phase/3-phase absolute transform. The symmetric pulses like Fig. 9 are output. It can be available to control precisely by (3). As below, the order of generation of the pulses is shown.

The area is decided by $\Delta T_{\alpha}$ and $\Delta T_{\beta}$ in Fig. 10. The order and the switching times $\sqrt{3/2} \Delta T_{i,j}$ of the output vectors $V_{i,j}$ in each areas are decided by Table. II and III. Here, coefficient $\sqrt{3/2}$ is the coefficient to transform two-phase into three-phase, and the output order is decided for the number of switching times to be fewer.

For example, the pulses of Fig. 9 are output in the area VI. In this case, the switching times which is $\Delta T_1 = \sqrt{3/2} \Delta T_i$ and $\Delta T_6 = \sqrt{3/2} \Delta T_j$ is shared as Table. III.

F. Design of Feedback Control

A proportional-integral-derivative position controller was designed, as band frequency of the position loop was 100 Hz. The sensitivity characteristic is shown in Fig. 11. The steady-state position error is shown in Fig. 12. The standard deviation $3\sigma$ of the steady-state position error is 59.7 nm.

IV. CONSIDERING RESONANCE MODE

Generally, the plant has resonance modes caused by pitching in positioning of a stage. In a large-scaled stage, a lot of resonance modes exist from several helz to several kilo helz. Moreover, most of the resonance modes have the antiresonance modes. Nano-stage has the primary resonance mode at 670 Hz and the antiresonance mode at 690 Hz in Fig. 2.

The characteristic $G_{vib}(s)$ of the primary resonance mode and the antiresonance mode is represented as

$$
G_{vib}(s) = \frac{0.9429s^2 + 32.53s + 17720000}{s^2 + 33.5s + 17720000}.
$$

(16)

It is the biproper minimum phase system. Therefore, the stable inverse model of $G_{vib}(s)$ can be designed. The inverse model is defined as the vibration suppression filter (VSF).

$$
VS F(z) = G_{vib}^{-1}(z)
$$

(17)

$VSF[z]$ which is discretized by shorter period $T_u$ with Prewarp-tustin transformation is applied in multirate PWM positional control system as Fig. 13. The resonance mode can be suppressed with the feedforward controller.

This method can suppress the resonance modes in just proportion because inverse model of the resonance modes is applied. It is not necessary to previously give target trajectories which does not have the primary vibration characteristic as MPVT (minimizing the primary vibration trajectory).

Moreover, PTC method assures output errors are perfectly zeros per longer period $T_e$ in ideal condition as the nominal plant. The period $T_e$ of the system applied VSF is shorter than one of vibration suppression PTC in [10] by $n_vT_u$ ($n_v$: number of considering resonance modes). It is very important in the case of positioning in several sampling periods.

As above, vibration suppression control can be achieved easily and efficiently by considering the antiresonance mode.

V. SIMULATIONS

In the specification of Table. IV, simulations of ultrahigh-speed nanoscale positioning are performed. In the table, $t_d$
is the positioning time, and $A^{\text{ref}}$ is the target position. The target position trajectory is based on a 5-order polynomials. Here, position error tolerance is defined as 100 nm. Multirate PWM position control was compared with original PTC based on the discrete model by zero-order hold.

Simulation results of Spec. 1 are shown in Fig. 14. In Spec. 1 of 10 ms positioning, all methods can achieve the position error tolerance. Especially, ‘Proposed’ is better in transient response.

Simulation results of Spec. 2 are shown in Fig. 15 and 16. In Spec. 2 of 2 ms positioning, all methods can achieve the position error tolerance as Spec. 1. Considering more precise plant model, the transient response is better. However, residual vibrations exist in all method because of resonanse mode at 670 Hz.

In Fig. 16 with VSF, good positionings can be achieved without residual vibrations as simulations. Moreover, ‘Proposed’ is better than ‘Conventional 2’ in Fig. 18 and 19 because of considering the PWM hold and the current loop. However, It seems that resonance modes influence positioning more than modeling errors in faster positioning.

**VI. CONCLUSIONS**

In this paper, we propose multirate PWM control considering resonance mode for more precise positioning. Simulations and experiments are performed to show the advantages of the proposed method. The proposed method can achieve vibration suppression control easily and efficiently by considering the antiresonance mode. In future, it seems that it is important to research the relation between machine characteristics and antiresonance characteristics on the design of precise stages.

**REFERENCES**


**TABLE IV**

<table>
<thead>
<tr>
<th></th>
<th>$T_u$</th>
<th>$T_y$</th>
<th>$\tau_d$</th>
<th>$A^{\text{ref}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spec. 1</td>
<td>0.1 ms</td>
<td>0.1 ms</td>
<td>10 ms</td>
<td>1 $\mu$m</td>
</tr>
<tr>
<td>Spec. 2</td>
<td>0.1 ms</td>
<td>0.1 ms</td>
<td>2 ms</td>
<td>1 $\mu$m</td>
</tr>
</tbody>
</table>
(a) Position response

(b) Positional error response

Fig. 14. Simulation results of Spec. 1.

Fig. 15. Simulation results 1 of Spec. 2.

Fig. 16. Simulation results 2 of Spec. 2.

(a) Position response

(b) Positional error response

Fig. 17. Experimental results of Spec. 1.

Fig. 18. Experimental results 1 of Spec. 2.

Fig. 19. Experimental results 2 of Spec. 2.