Torque Ripple Suppression Control for PM Motor with Current Control Based on PTC

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Abstract—PM motor drive systems are widely used for industrial drives. However, PM motors basically produce the torque ripple due to the fluctuation of the magnetic field distribution. Dead time of the inverter, offset of sensors and current measurement errors lead to the torque ripple, too. The torque ripple leads vibration noises. In this paper, we proposed a torque ripple suppression method based on perfect tracking control, in which the torque ripple is measured by a low-bandwidth sensor. Finally, we show the advantages of the proposed method by simulations and experiments with a SPMSM.

Index Terms—PMSM, perfect tracking control, speed variation, suppression of torque ripple, current dependence

I. INTRODUCTION

Recently, permanent magnet (PM) motor drive systems are widely used for industrial drives and automotive applications. However, in the case of surface permanent magnet synchronous motor (SPMSM), an imperfect sinusoidal flux distribution causes the torque ripple. In the case of interior permanent magnet synchronous motor (IPMSM), the motor structure and a placement of permanent magnet generates the torque ripple. Moreover, the dead time of the inverter, the offset of sensors and current measurement error causes the torque ripple. The torque ripple leads vibration and noises. Thus, it is necessary to suppress the torque ripple.

Many researches and developments have been done on harmonic current and torque ripple suppression. In [1], harmonic current suppression method has proposed in rotating coordinate which is synchronized with the harmonic component for IPMSM. In addition, torque ripple suppression method was developed with plant model including harmonic current for the IPMSM which has distorted induced voltage in [2]. Moreover, torque ripple suppression control method which is modeling the torque ripple using motor position with feedforward compensation has proposed in [3]. In [4], cogging torque compensation method of the linear motor with adaptive robust control has proposed. Furthermore, torque ripple suppression method of brushless DC motor with direct torque control has proposed in [5]. In [6], the practical design considerations of a low torque ripple PM motor drive for electric power steering (EPS) application has proposed.

Our research group proposed the torque ripple suppression control method with high resolution encoder in [7]. In [8], the torque ripple suppression method in which the torque ripple is measured by a high-bandwidth torque meter is developed. In this paper, torque ripple suppression control method with speed variation is proposed. The torque ripple depends on the rotor angle. Therefore, the disturbance caused by the flux distortion can be described as the function of the rotor angle. Moreover, the current relativity of the torque ripple is considered. Torque ripple is proportional to current fundamental component $i_o(t)$. Finally, we show the advantages of the proposed method by simulations and experiments.

II. MODELING OF SYSTEM

A. dq Model of IPMSM

The voltage equation is described as

$$
\begin{bmatrix}
    v_d \\
    v_q 
\end{bmatrix} =
\begin{bmatrix}
    L_{ds} + R & -\omega L_q \\
    \omega L_d & L_{qs} + R
\end{bmatrix}
\begin{bmatrix}
    i_d \\
    i_q
\end{bmatrix} + \omega K_e \begin{bmatrix}
    0 \\
    1
\end{bmatrix},$
$$

(1)
where \(v_{d(q)}\) is the \(d(q)\) axis stator voltage, \(R\) is the winding resistance, \(L_{d(q)}\) is the \(d(q)\) axis winding inductance, \(\omega\) is the angular electric velocity, \(i_{d(q)}\) is the \(d(q)\) axis stator current, \(K_e\) is the flux linkage, respectively. In the case of IPMSM, \(L_d\neq L_q\). So, the relation between torque \(T\) and \(\omega\) is represented as
\[
\omega = \frac{1}{Js + \beta} T, \quad T = K_t i_q + K_r i_{d(q)} \quad (2)
\]
where \(K_t\) is the torque constant, \(K_r = P(L_d - L_q)\) and \(P\) is the number of poles. Therefore, the \(dq\) coordinate model of IPMSM is shown in Fig. 1. In this paper, \(d\) axis current is controlled to 0. So reluctance torque is ignored.

The decoupling control (3) is applied to the plant (1) as follows.
\[
v_d = v'_d - \omega L i_q, \quad v_q = v'_q + \omega (L_d i_d + K_e), \quad (3)
\]
where \(L = L_q\).

The state equation of the continuous-time system is represented as
\[
\dot{x}(t) = A_c x(t) + B_c u(t), \quad y(t) = C_c x(t) \quad (4)
\]
\[
A_c = -\frac{R}{L}, \quad B_c = \frac{1}{L}, \quad C_c = 1,
\]
where \(x\) is the \(d(q)\) axis current, \(u\) is the \(d(q)\) axis voltage, respectively.

III. CONTROL SYSTEM DESIGN

A. Measurement Principle of Torque Meter

Meseasurement principle of torque meter is explained. In this method, the torque meter is installed between the test motor and the load motor. It can be interpreted as two-inertia system. The block diagram is shown in Fig. 2[9]. Thus, it is possible to use this model to detect torque ripple exactly with torque meter.

B. Discretization of Plant

In general, the discrete-time model of the controlled plant is obtained with the zero-order-hold. The zero-order-hold discretization approximately assumes that the inverter can output the arbitrary voltage. In the case of controlling instantaneous values precisely, it is unsuitable. The single-phase inverter output can take only 0 or \(\pm E\). The switches are turned on and off once during each interval \(T_u\) such that a voltage pulse of magnitude \(E(0 - E)\) and width \(\Delta T\) centered in the interval \(T_u\). Then, this paper uses PWM hold model which can evaluate the instantaneous voltage precisely by regarding pulse width as control input. According to [10], the plant of the inverter drive system can be discretized based on the PWM hold, the discrete time model of the plant is derived by
\[
x[k + 1] = A_s x[k] + B_s \Delta T[k], \quad y[k] = C_s x[k] \quad (5)
\]
\[
A_s = e^{A_s T_u}, \quad B_s = e^{A_s T_u / 2} B_s E, \quad C_s = C_c.
\]
The control input of the proposed method is the switching times \(\Delta T_d\) and \(\Delta T_q\) because the control system is designed by the \(dq\)-model. In order to apply the PWM hold model to PM motor, the input for PWM pulse of the three-phase system has to be generated[11].

C. Proposed Method

Fig. 3 shows the structure of the proposed method. The proposed method consists of the perfect tracking control (PTC)[12] and the periodic signal generator (PSG). PTC consists of the 2-degree-of-freedom control system as shown in Fig. 3. The feedforward controller is a stable inverse system of the plant, and assures the perfect tracking for a nominal plant at the sample point. Because the plant is the first order system, PTC is achieved by a usual singlerate control. The design of the feedforward controller is described. By discretizing (5) with PWM hold model, (7) is obtained.
\[
x[k + 1] = A x[k] + B u[k], \quad y[k] = C x[k] \quad (6)
\]
\[
A = e^{-\frac{2}{T_s} T_u}, \quad B = e^{-\frac{2}{T_s} T_u} \frac{1}{L} V_{dc}, \quad C = 1.
\]
Therefore, a stable inverse system of the plant is obtained as (7), and a nominal output is given as (8).
\[
u_0[k] = B^{-1}(1 - z^{-1} A)x_d[k + 1], \quad (7)
\]
\[
y_0[k] = z^{-1} C x_d[k + 1]. \quad (8)
\]
The proposed method applies PSG to the PTC method. In the proposed method, the switch 1 is turned on during one disturbance period \(T_d\) in the steady-state. After the data is stored in the memory, switch 1 is turned off and switch 2 turned on. In this case, the disturbance is equal to the torque ripple. Therefore, these compensations are purely feedforward because they are injected after the switch 1 turns off. When the disturbance period and control period are \(T_d\) and \(T_s\), the number of memories \(N_d\) becomes \(N_d = T_d / T_s\). The \(N_d\) data give the compensation signal and the one-sample ahead reference signal is generated by using the PSG in Fig. 3. PSG gives an appropriate compensation signal as long as disturbance does not change suddenly, and can suppress the periodic disturbance at every sample point.

D. Generation Method of Compensation Signals

Generation method of compensation signals is explained. The torque generated by PMSM is described as
\[
T_M(t) = \frac{1}{2} i_s(t) \frac{d\Psi_{ss}(\theta)}{d\theta} + i_s(t) \frac{d\Psi_{sr}(\theta)}{d\theta} + \frac{1}{2} i_r(t) \frac{d\Psi_{rr}(\theta)}{d\theta}, \quad (9)
\]
where $i_s$ is magnetomotive force of stator, $i_r$ is magnetomotive force of rotor, $\Psi_{ss}$ is the flux linkage excited by $i_s$ and interlinking with stator winding, $\Psi_{sr}$ is the flux linkage excited by $i_r$ and interlinking with stator winding, $\Psi_{rr}$ is the flux linkage excited by $i_r$ and interlinking with rotor [13]. The first term of (9) is reluctance torque. The second term is main torque and the third term is cogging torque. Now, we define $d\Psi_{sr}(\theta)/d\theta$ as $\Psi'(\theta)$ and suppose that current of winding and flux have fundamental components and harmonic components each in. Moreover, torque components generated by the fundamental component $\Psi'_o$ and the fundamental current $i_o$ is defined as $T_{Mo}$. The torque component generated by harmonic current $i_h$, harmonic flux $\Psi'_h$ and cogging torque is also defined as $T_r(\theta, i)$ with parameters of position $\theta$ and current $i$. (9) can be represented as,

$$T_M(t) = T_{Mo}(t) + T_r(\theta, i),$$

$$T_{Mo}(t) = i_o(\theta)\Psi'_o,$$

$$T_r(\theta, i) = i_h(\theta)\Psi'_h(\theta) + i_o(\theta)\Psi'_h(\theta) + i_h(\theta)\Psi'_h(\theta) + T_{cog}(\theta),$$

where $T_{cog}(\theta)$ is the cogging torque and suffix o refers fundamental components, suffix h refers harmonic components. Therefore, the motion equation of rotation system about motor including torque ripple is represented as

$$J_M\dot{\omega}(t) = K_i i(t) + T_r(\theta, i) - B_M \omega(t) - T_{ev}(t).$$

$$K_i i(t) = \Psi'_o(i_o(t) + i_r(t)),$$

where $T_r$ is the torque of a shaft torsion and suppose that this torque can be detected from torque meter. $i_o(t)$ is the current additionally generated by PTC after the switch 2 turns on. When the switch 1 turns on, i.e., $i_r(t) = 0$, torque ripple generated in motor can be estimated by

$$\dot{T}_r(\theta, i) = T_u(t) - K_i i(t) + B_M \omega(t) + J_M\dot{\omega}(t).$$

$\dot{T}_r(\theta, i)$ calculated by (12) is stored in the memory as data of periodic disturbance. The current reference which suppresses $\dot{T}_r(\theta, i)$ by $K_i i(t)$ and desired torque $T_{Mo}(t)$ is generated as (13).

$$i^*(t) = i_{ref}(t) - i'_r(t)$$

$$i_{ref}(t) = \frac{1}{K_i} T_{Mo}(t),$$

IV. COMPENSATION METHOD WITH SPEED VARIATION[14]

In [15], when the motor speed changes, the harmonic components are suppressed with the compensation signal which is obtained by relearning after the speed variation. However, it takes time for the relearning and the disturbance during the transient state is suppressed only by the PI control. Therefore, the novel method which does not need relearning is proposed. In (10), $T_r(\theta, i)$ can be described as (14) if $i_h(t)$ becomes perfectly zero by the dead time compensation.

$$T_r(\theta, i) = i_o(\theta)\Psi'_h(\theta) + T_{cog}(\theta)$$

In this equation, it assumes that the cause of the torque ripple is flux distribution. This distribution depends on the rotor angle. Therefore, the torque ripple caused by the flux distortion can be described as the function of the rotor angle. For example, 6th-order-components on driving frequency 10 [Hz] is the same as that on 20 [Hz]. We can suppress the torque ripple by saving its information to the memory and generating the compensation signal which corresponds to the rotor position. Therefore, the torque ripple is suppressed not only after the speed variation but also during the speed variation.

V. RELATIVITY BETWEEN CURRENT AND THE TORQUE RIPPLE

In this section, relativity between current and the torque ripple is verified. In the case that cogging torque $T_{cog}(\theta)$ is small enough to ignore, (14) is represented as (15).

$$T_r(\theta, i) \simeq i_o(\theta)\Psi'_h(\theta)$$

This equation represents that $T_r(\theta, i)$ is described by fundamental component of current $i_o(t)$ and harmonic component of flux $\Psi'_h$ in the case that cogging torque is small enough to ignore. Therefore, the torque ripple is considered to be rise and fall proportional to fundamental component of current $i_o(t)$. If $i_o(t)$ is changed, torque ripple can suppress without re-learning with proportionality factor based on current value.

VI. SIMULATION

The torque ripple suppression simulation with speed variation was carried out to confirm the effectiveness of the proposed method. In this simulation, the torque ripple includes 6th-order component and 12th-order component. These torque ripple components are detected by the experiment. The motor speed is controlled by the load motor at 10 [Hz] and learning the torque ripple. The torque ripple compensation begins from 2.0 [s]. After that, the motor speed changes to 20 [Hz]. We start the torque ripple suppression control by compensation signals learning at 10 [Hz] from 4.0[s]. The results of simulation are shown in Fig.5. Fig.5 (a) and (b) show the waveform of $T_{ev}$. Fig.5 (c) and (d) show the spectrum of the $T_{ev}$ at 10 [Hz] and (e) and (f) show the spectrum of the $T_{ev}$ at 20 [Hz]. These figures show that the torque ripple suppression control with speed variation without re-learning.
Here, $q$ axis current waveform at 10 [Hz] is shown in Fig.5 (g). $q$ axis current at 20 [Hz] is shown in Fig.5 (h). Moreover, its spectrums are shown in Fig.5 (i) $\sim$ (j). As shown in Fig.5 (g) $\sim$ (j), PSG outputs the compensation signals to suppress the torque ripple without re-learning.

### VII. EXPERIMENT

#### A. Speed Variation

The torque ripple suppression experiment was carried out by using IPMSM to confirm the effectiveness of the proposed method. The specification of the IPMSM which is used in the experiment is shown in Table I. The load motor is driven by the speed control mode and controls the motor speed. The test motor is driven by the torque control mode. First, PSG is learning the torque ripple at 10 [Hz] of the motor speed. After the learning, the motor speed changes to 20 [Hz]. After that, the torque ripple compensation starts. When compensation signal is generated in the proposed method, the averaging is done the two type of the torque ripple suppression control. In the case1, the torque ripple is learning at 16 [A] of $i_o(t)$. In the case2, the torque ripple is learning at 12 [A] of $i_o(t)$. After learning, $i_o(t)$ is changed to 16 [A]. We compared two method.

Fig.7 (a) shows the waveform of the $T_{ev}$ before compensation and (d) shows its spectrum. Fig.7 (b) and (c) show the waveform of the $T_{ev}$ after compensation and (e) and (f) show the spectrum of them. In the case of (b) and (e), compensation signals are generated by 16 [A] of $i_o(t)$ and in the case of (c) and (f), compensation signals are generated at 12 [A] of $i_o(t)$. Fig.7 (a) $\sim$ (f) show that each method can suppress the torque ripple at the same level. Therefore, the torque ripple is proportional to $i_o(t)$ in this condition. Fig.7 (g) $\sim$ (l) show the results of $q$ axis current. They prove that PSG output the almost same compensation signals to suppress the torque ripple with proportionality factor based on current value.

### VIII. CONCLUSION

In this paper, the torque ripple suppression control method with speed variation and relativity between current and torque ripple were discussed. It is proved that the cause of the torque ripple is flux distribution and the torque ripple caused by the flux distortion can be described as the function of the rotor angle. In the simulation and experiment, we prove that the torque ripple can suppress with speed variation without re-learning.

Moreover, the torque ripple is described by fundamental component of current $i_o(t)$ and harmonic component of flux $\Psi'$. Therefore, the torque ripple is considered to be rise and fall proportional to fundamental component of current $i_o(t)$. In the simulation and experiment, we prove that the torque ripple can be suppressed at the same level with or without re-learning by proportionality factor based on current value.

### REFERENCES


Fig. 5. Simulation results.

(a) Waveform of $T_{ev}(10\text{Hz})$. (b) Waveform of $T_{ev}(20\text{Hz})$.

(c) FFT of $T_{ev}$(before compensation, 10Hz). (d) FFT of $T_{ev}$(after compensation, 10Hz).

(e) FFT of $T_{ev}$(before compensation, 20Hz). (f) FFT of $T_{ev}$(after compensation, 20Hz).

(g) Waveform of $i_q$(10Hz). (h) Waveform of $i_q$(20Hz).

(i) FFT of $i_q$(before compensation). (j) FFT of $i_q$(after compensation, 10Hz).

(k) FFT of $i_q$(after compensation, 20Hz).

Fig. 6. Experimental results of torque ripple suppression with speed variation.

(a) Waveform of $T_{ev}(20[\text{Hz}])$, before compensation. (b) Waveform of $T_{ev}(20[\text{Hz}])$, after compensation.

(c) FFT of $T_{ev}(20[\text{Hz}])$, before compensation. (d) FFT of $T_{ev}(20[\text{Hz}])$, after compensation.

(e) Waveform of $q$ axis current(20[Hz], before compensation). (f) Waveform of $q$ axis current(20[Hz], after compensation).

(g) FFT of $q$ axis current(20[Hz], before compensation). (h) FFT of $q$ axis current(20[Hz], after compensation).
Fig. 7. Experimental results of torque ripple suppression control.