Abstract—The switching loss of the inverter depends on the inverter carrier frequency. Therefore, the high carrier frequency is unfavorable for electric vehicles. However, the low carrier frequency decreases the slip ratio control performance. To achieve the high-speed current response with the low carrier frequency, it is required to consider the inverter dynamics. Then in this paper, we propose a slip ratio control method with single-rate PWM considering driving force with low carrier frequency. Using this method, we can achieve the high responsive slip ratio control. Simulations and experiments are carried out to demonstrate the effectiveness with EVs.

I. INTRODUCTION

Recently, electric vehicles (EVs) have attracted a great deal of interest as a solution of environment concern. The advantages of EVs are not only good for environment but also for the high motion control performance. From a point of view of control engineering, the greatest characteristic of EVs is the capability of motion control offered by using the electric motor. Compared with internal combustion engines, electric motors have advantages such as the fast torque response, ease of measuring the motor torque, and the possibility of being attached in each wheel. By making effective use of these advantages, we can achieve the advanced vehicle motion control which is impossible to be achieved by internal combustion engines. Many researches into the vehicle motion control have been conducted such as [1], [2], [3]. In the paper [4], the skid prevention control of EVs considering the motor dynamics has been proposed. Our research group proposed the traction control which can prevent wheel slip on slippery roads for EVs which has the in-wheel motors [5], [6], [7].

In general, higher inverter switching frequency increases switching losses. Improving cruising distance with a single charge is a critical issue of EVs, so increase of losses is undesirable. Therefore the electric motor of EVs or HEVs (Hybrid Electric Vehicles) on market is driven with the low carrier frequency [8]. However, the low carrier frequency decreases the performance of the feedback controller and the slip ratio control.

In this paper, we propose the slip ratio control of EVs with single-rate PWM (Pulse Width Modulation) with consideration of the driving force. By controlling the slip ratio with the feedforward controller which is based on the vehicle model with the driving force, the slip ratio control system does not depend on the feedback controller. Even if the performance of the current feedback controller is worsen, the slip ratio control system can achieve the fast response. The performance of the proposed method is shown by simulations and experiments.

II. VEHICLE DYNAMIC MODEL

In this chapter, the longitudinal vehicle motion dynamics is shown. Using one wheel model, as shown in Fig. 1, the longitudinal motion equations of the wheel and the vehicle chassis can be given as

\[ \begin{align*}
J\dot{\omega} &= T - rF_d \\
MV &= F_d,
\end{align*} \]

(1)

(2)

where \( \omega [\text{rad/s}] \) is the wheel speed, \( V [\text{m/s}] \) is the vehicle speed, \( T [\text{Nm}] \) is the motor torque, \( F_d [\text{N}] \) is the driving force, \( M [\text{kg}] \) is the vehicle mass per drive wheel, \( r [\text{m}] \) is the wheel radius and \( J [\text{Nm}^2] \) is the wheel inertia. When the vehicle model is two-wheel drive, the vehicle mass per drive wheel \( M \) is given as

\[ M = \frac{M_0}{2}, \]

(3)
where \( M_0 [\text{kg}] \) is the vehicle mass. The slip ratio \( \lambda \) is defined as
\[
\lambda = \frac{V_ω - V}{\max(V_ω, V, \varepsilon)}.
\]
where \( \varepsilon \ll 1 \) is the small constant to avoid zero denominator. Because this paper deals with the acceleration only, we assume that \( \max(V_ω, V, \varepsilon) = V_ω \) all the time. In this parer, the slip ratio is defined as
\[
\lambda = \frac{V_ω - V}{V_ω}.
\]
In the simulation, Magic Formula [9] is adopted as the model between the friction coefficient \( \mu \) and \( \lambda \). \( F_d \) is defined as
\[
F_d = \mu N,
\]
where \( N \) is the normal force per wheel. \( N \) is defined as
\[
N = \frac{M_0 g}{4},
\]
where \( g \) is the gravity acceleration.

We assume that the in-wheel motor is the surface permanent magnet synchronous motor (SPMSM). The circuit equation of q-axis of SPMSM given as
\[
v = Ri + L \frac{di}{dt} + K_e ω,
\]
where \( v [\text{V}] \) is the armature voltage, \( R[\Omega] \) is the armature resistance, \( L[\text{H}] \) is the inductance, \( i[\text{A}] \) is the armature current and \( K_e [\text{Vs/rad}] \) is the back EMF constant. The torque \( T[\text{Nm}] \) is given as
\[
T = K_t i,
\]
where \( K_t [\text{Nm/A}] \) is the torque constant. The block diagram of the one wheel EV model with the motor model is obtained as Fig. 2.

III. WHEEL SPEED CONTROL WITH MULTIRATE PWM AND PTC

A. PWM Hold Model [10][11]

In this chapter, we explain PWM hold model with the single-phase inverter. However, the three-phase inverter is employed to drive the motor. The single-phase inverter can be handled same as the three-phase inverter by using space vector modulation[10].

The single-phase inverter can output only 0 or \( \pm E [\text{V}] \) as Fig. 3. Therefore in the case of controlling instantaneous values precisely, discretization of the plant model by zero order hold is unsuitable. So we treat the output of the inverter as PWM hold. The block diagram of the one wheel EV model with the motor model is obtained as Fig. 4

\[
x[k+1] = A_s x[k] + B_s u[k], y[k] = C_s x[k]
\]
\[
A_s = e^{A_s T_s}, B_s = e^{A_s T_s/2} B_s E, C_s = C_e.
\]

When \( \Delta T[k] \) is negative, \( -E [\text{V}] \) is output.

B. Perfect Tracking Control (PTC)[12]

PTC applies the multirate feedforward controller which is the stable inverse system of the plant, where the references are state variables. Therefore, the perfect tracking is guaranteed on the sampling period of the reference \( T_r \). PTC is shown in Fig. 6. In the multirate feedforward control, there are three sampling periods \( T_r, T_y \), and \( T_u \) which represent the periods of the reference signal \( r(t) \), the output \( y(t) \), and the control input \( u(t) \), respectively. The control input \( u(t) \) is changed \( n \) times during one sampling period of the reference \( T_r \) as shown in Fig. 5. Relationships between each period are given as
\[
\frac{T_r}{n} = T_y = T_u,
\]
where \( n \) is the plant order. The coefficient matrices \( A, B, C \), and \( D \) derived as (15) from the plant model discretized by \( T_u \).

\[
x[k+1] = A_s x[k] + B_s u[k], y[k] = C_s x[k]
\]
\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_s & 0 & \cdots & 0 \\ c_s A_s & c_s B_s & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots \\ c_s A_s^{n-1} & c_s A_s^{n-2} B_s & \cdots & c_s B_s \\ 0 \end{bmatrix}.
\]

PTC consists of the 2DOF control system. When the tracking error is occured by the modeling error or the disturbance, it can be suppressed by the feedback control.

IV. CONTROL SYSTEM DESIGN

A. Slip Ratio Control with Multirate PWM and PTC

In this section, we explain slip ratio control with multirate PWM and PTC as the previous proposed method [7]. The
transfer function from the input voltage \( v \) to the wheel speed \( \omega \) is described by

\[
\frac{\omega}{v} = \frac{K_i}{JLs^2 + JRs + K_eK_i}.
\] (16)

The controllable canonical form of (16) with the state variables \( \mathbf{x} = [\omega \ \dot{\omega}]^T \) is represented by

\[
\begin{align*}
\dot{x}(t) &= A_c x(t) + b_c u(t) \\
y(t) &= c_c x(t)
\end{align*}
\] (17)

\[
\begin{bmatrix}
A_c & b_c \\
c_c & 0
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
-K_eK_i & -JRs & K_e \\
1 & 0 & 0
\end{bmatrix}.
\] (19)

By discretizing (17) and (18) with the sampling period \( T_u \) based on (11) and (12) as the input is the switching time \( \Delta T \), the matrices \( A, B, C, \) and \( D \) are derived by (15).

The transfer function from \( v \) to the current \( i \) is described by

\[
\frac{i}{v} = \frac{Js}{JLs^2 + JRs + K_eK_i}.
\] (20)

Therefore, the controllable canonical form of (20) with the state variables \( \mathbf{x} = [\omega \ \dot{\omega}]^T \) is represented by

\[
\begin{align*}
\dot{x}(t) &= A'_c x(t) + b'_c u(t) \\
y(t) &= c'_c x(t)
\end{align*}
\] (21)

\[
\begin{bmatrix}
A'_c & b'_c \\
c'_c & 0
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
-K_eK_i & -JRs & K_e \\
1 & 0 & 0
\end{bmatrix}.
\] (23)

In order to obtain the nominal current \( i_0 \), the coefficient matrices of the current multirate feedforward controller based on (21) and (22) \( A', B', C', \) and \( D' \) are required. By discretizing (21) and (22) in the same way that discretizes (17) and (18), the matrices \( A', B', C', \) and \( D' \) are derived by (15). Because \( A_c = A'_c \) and \( b_c = b'_c \ c'_c \), \( A = A' \) and \( B = B' \).

As stated above, the multirate feedforward controller can be designed which is the stable inverse system of \( q \) axis of SPMISI. The output of this feedforward controller is the switching time \( \Delta T \) as the control input \( u_0 \). The slip ratio control system is shown in Fig. 7. Where \( C_I \) is the current controller and \( C_\omega \) is the wheel speed controller.

\[
C_i \text{ is the PI controller which is designed by pole-zero cancellation. The plant and the controller are given as}
\]

\[
i = \frac{1}{Ls + R} v
\] (24)

\[
C_i(s) = \frac{Ls + R}{\tau_i s}.
\] (25)

\[
C_\omega \text{ is the PI controller which is designed by pole placement method. The plant and the controller are given as}
\]

\[
\omega = \frac{1}{J_s T} \frac{T}{K_i s + K_i}
\] (26)

\[
C_\omega(s) = \frac{J_s}{K_i s + K_i}
\] (27)

where this plant ignores the driving force.

From (5), the reference of the wheel speed which is calculated from the target slip ratio \( \lambda^* \) is given as

\[
\omega^*[i] = \frac{V[i]}{r(1 - \lambda^*[i])}.
\] (28)

**B. Slip Ratio Control with Single Rate PWM Considering Driving Force**

In this section, novel slip ratio control is proposed based on single-rate PWM and PTC considering driving force. By differentiating (4) with respect to the time, the following equation is obtained.

\[
r\omega\dot{\lambda} = r\dot{\omega}(1 - \lambda) - \dot{V}.
\] (29)
From (1) and (2), the following equation is obtained
\[ \dot{V} = \frac{T}{rM}. \tag{30} \]
From (29) and (30), the detailed motion equation which includes the time-derivative term of \( \lambda \) is derived as
\[ (J + r^2 M(1 - \lambda_n))\dot{\omega} - r^2 M\dot{\lambda}_n\omega = T. \tag{31} \]
Here, we assume \( \lambda \) and \( \dot{\lambda} \) as the sum of the nominal values \((\lambda_n, \dot{\lambda}_n)\) and the error components \((\Delta \lambda, \Delta \dot{\lambda})\).

\[ \begin{align*}
\lambda &= \lambda_n + \Delta \lambda \\
\dot{\lambda} &= \dot{\lambda}_n + \Delta \dot{\lambda}.
\end{align*} \tag{32} \tag{33} \]
(31) can be rewritten as
\[ (J + r^2 M(1 - \lambda_n))\dot{\omega} - r^2 M\dot{\lambda}_n\omega = T + r^2 M(\Delta \lambda \dot{\omega} + \Delta \dot{\lambda} \omega). \tag{34} \]
Extract the terms of \( \lambda_n \) and \( \dot{\lambda}_n \) from (34), the transfer function from the torque to the wheel speed which considers only the nominal values is given as
\[ \frac{\omega}{T} = \frac{1}{(J + r^2 M(1 - \lambda_n))s - r^2 M\dot{\lambda}_n}. \tag{35} \]
Here, we assume \( \dot{\lambda}_n = 0 \), the nominal plant is given as
\[ \frac{\omega}{T} = \frac{1}{J ns}, \tag{36} \]
where \( J_n \) is
\[ J_n = J + r^2 M(1 - \lambda_n). \tag{37} \]
The ignored terms of the error components behave as the disturbances for the nominal plant. So, the reference is the sum of the value \( i_n \) which can attain the reference of the wheel acceleration for the nominal plant and the value \( i_{dec} \) which can cancel the terms of the error components. From (36) \( i_n \) and \( i_{dec} \) given as
\[ i_{dec} = -\frac{r^2 M(\Delta \lambda \dot{\omega} + \Delta \dot{\lambda} \omega)}{K_I}, \tag{38} \]
\[ i_n = \frac{J_n}{K_I} \dot{\omega}^*. \tag{39} \]
The current is controlled by the single-rate feedforward controller which is based on PTC. The current feedforward controller can be designed by the same way as the multirate feedforward controller. The transfer function from the input voltage to the current is described by
\[ \frac{i}{v} = \frac{1}{Ls + R}. \tag{40} \]
The state-space representation of (40) with the state variable \( x = i \) is represented by
\[ \begin{align*}
\dot{x}(t) &= A_x x(t) + b_c u(t) \tag{41} \\
y(t) &= c_c x(t) \tag{42} \\
\begin{bmatrix} A_x \\ b_c \\ c_c \end{bmatrix} &= \begin{bmatrix} -\frac{R}{T} & 1 \\ 1 & 0 \end{bmatrix}. \tag{43}
\end{align*} \]
Discretize (41) and (42) by the sampling period \( T_s \) based on (11) and (12) as the input is the switching time \( \Delta T \), the matrices \( A, B, C, \) and \( D \) are derived by (15).

As stated above, the current single-rate feedforward controller can be designed. The slip ratio control system considering the driving force is shown in Fig. 8. Where \( C_I \) is the current controller and \( C_o \) is the wheel speed controller.

\( C_I \) is the PI controller which is designed by pole-zero cancellation base on the plant (40). \( C_o \) is the PI controller which is designed by pole placement method based on the plant (36).

V. SIMULATION

The tracking responses of the slip ratio of each control method are compared in the simulations. The road condition is assumed to be low \( \mu \) road surface with \( \mu_{max} = 0.2 \). The model between \( \mu \) and \( \lambda \) is shown in Fig. 9. The PWM inverter are utilized with 2[kHz] carrier frequency. Therefore, each sampling period are \( T_u = T_y = T_r/2 = 500[\mu s] \) in the multirate feedforward control and \( T_u = T_y = T_r = 500[\mu s] \) in the single-rate feedforward control. \( C_o \) is designed so that the poles of the wheel speed control loop are located at -20[rad/s]. \( C_I \) is designed so that the time constant of the current control loop is 5[ms]. The vehicle speed \( V \) is assumed to be detectable. The specification of the EV is shown in Table. I.

Fig. 11, 12, and 13 show the simulation results of the conventional method shown in Fig. 10, the previous proposed method and the new proposed method, respectively. Fig. 11(a) shows the slip ratio response with the conventional method. The response speed is slow because of the slow pole of the speed control loop. Fig. 12(a) shows the slip ratio response of the previous proposed method. The response speed is much faster than the one of the conventional method. Moreover the tracking error is small. The slip ratio response has the overshoot caused by the zero of feedback controller which suppresses the driving force. Fig. 13(a) shows the slip ratio response with the new proposed method. The response speed is fast, and the tracking error is little. The slip ratio response has very little overshoot which is smaller than the one of the previous proposed method.

VI. EXPERIMENT

The tracking responses of the slip ratio of each control method are compared in the experiments. In the experiments,
the vehicle runs on the slippery boards. The experiment condition is same as the simulation. The vehicle speed $V$ is measured by the non-drive wheel speed.

Fig. 15 shows the experimental result with the conventional method. The large tracking error arises due to the slow response. Fig. 16 and 17 show the result with the previous proposed method and the new proposed method, respectively. The ripple of the responses is due to the low resolution angular speed sensor. By using the feedforward controller, the fast slip ratio response is achieved.

**VII. CONCLUSION**

In this paper, the slip ratio control method with the feedforward controller considering driving force was proposed.
Effectiveness of the new proposed method was confirmed by the simulation and the experiment. By using the model which considers the driving force for design the feedforward controller, the new proposed method can control slip ratio more precise than the previous proposed method whose feedforward controller was designed without consideration of the driving force.

The future work is to compensate the variation of the vehicle mass which causes the modeling error.

REFERENCES


