Positioning Control for Piezo Scanner using Multirate Perfect Inverse Model Based Iterative Learning Control

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Abstract—Recently, in high precision positioning field, the improvement of the positioning accuracy is required for the development of the next generation technology like nanotechnology. In many industry application machines, we can often see the repetitive operation [7]. For the repetitive positioning command of the positioning machine, the investigation for the suppression of the repetitive tracking error using iterative learning control (ILC) is increasing. For above reasons, the purpose of this paper is improvement of the positioning accuracy in the high-speed repetitive position command. The plant is the Piezo-scanner of the atomic force microscope which is not only used for measurement device but nano-manipulation, the requirement of positioning accuracy of the AFM is nano-scale. In ILC, the inverse system of the plant is often used as the learning filter. This design of the learning filter using the inverse system is most important part in ILC system design. Thus, there are many methods of the inverse system design. In this paper, the ILC using the perfect inverse system of the plant which based on the perfect tracking control is proposed. Its proposed inverse system of the plant is the perfect inverse system of the plant, and the all poles of the proposed inverse system of the plant can be located on the origin in discrete-time domain. Thus, the proposed inverse plant has good characteristics for ILC system. In this paper, the effectiveness of the proposed method is shown by the some simulations and experimental results.

I. INTRODUCTION

The positioning accuracy required of a precision positioning device was getting on improving, and required scale has changed from the microscale to nanoscale. In this paper, the piezo scanner of the atomic force microscope (AFM) is positioned at high speed on nano-scale. Usually, AFM is used as a device which measures nanoscale surface topography. However, it is utilized also for the nanoscale manipulation using the probe not only as the measuring device. AFM in this paper has the XYZ scanner of the piezo-electric element. In measurement of surface topography, X scanner is repetition operation. High accuracy is demanded in order that the positioning accuracy of the piezo scanner may affect the measurement accuracy of AFM directly. As pioneering investigation of the fast positioning control of XY scanner, the hardware of the piezo scanner is improved [2] or there are Zero Phase Error Tracking Control (ZPETC) and Zero Magnitude Error Tracking Control (ZMETC) which are the control-system designs which used the inverse system [3][4][5]. Iterative learning Control (ILC) is known by one of the ways of suppressing the tracking error to repetitive trajectory [6][7]. ILC is used for positioning of the machine tool and the semiconductor photolithography machine, and is the effective method in the field of fast positioning. This paper applies ILC to the piezo scanner of AFM, and aims at fast highly precise positioning in nanoscale. In ILC, using the inverse system of the plant for the learning filter is recommended, and the stability and error repressed difference appears with the construction of the learning filter. Moreover, in actual iteration learning control, since the control system becomes unstable easily, the learning gain is set as the small value and the method of suppressing the error gradually is used. This paper aims at error suppression in the short time by applying the stable multi rate inverse system derived by Perfect Tracking Control (PTC) [8] to the learning filter. Moreover, the adequate controller is designed from the convergence condition of ILC. Finally, simulation and the experiment show the effectiveness of the proposal method.

II. IDENTIFICATION OF THE PLANT

Fig. 1 shows the simplified structure of the plant which hardware is the commercial AFM (JSPM-5200, JEOL Ltd.). The position of Z-scanner can measure by the photo diode which detects the reflected laser beam from cantilever tip. The frequency characteristics of the plant Plant $P_s(z)$ and the nominal plant $P_n(z)$ are shown in Fig. ?? which is obtained by the servo analyzer. The transfer function of the continuous-time domain and the discrete-time domain for the plant $P_n(s)$, $P_n[z]$ are represented by eq.(1) and (2). Here, $P_n[z]$ is discretized with the zero-order hold at the
sampling time $T_s = 40[\mu s]$.

$$P_n(s) = \frac{5.24 \times 10^{13}}{s^3 + 2.60 \times 10^4 s^2 + 1.34 \times 10^9 s + 3.08 \times 10^3}$$

$$P_n[z] = \frac{391.86 \times 10^{-3} (z + 2.598) (z + 0.2326)}{z^3 - 0.6404 z^2 + 1.016 z - 0.353} \quad (2)$$

From eq.(2), the discretized nominal plant $P_n[z]$ has the unstable-zero $-2.598$.

III. CONTROL SYSTEM DESIGN

A. Introduction of ILC

At first, the signal used by ILC of this paper is explained. The following discrete time system which is discretized at the sampling time $T_s$, linearity time invariant system.

$$y_j[k] = G[q] u_j[k] \quad (3)$$

where $k$ shows $k = kT_s$ and $j$ shows the iterative number $(j = 1, 2, \cdots)$. The transfer function $G$ is the stable system. $q$ is the time shift operator. For example, $q(u_j[k])$ expresses $u_j[k+1]$. Here, if the iterative period $T_{ite}$ is $T_{ite} = NT_s$, the output $y_j$ and the control input $u_j$ at the iterative number $j$ is represented by

$$y_j = \begin{bmatrix} y_j[0] & y_j[1] & \cdots & y_j[N] \end{bmatrix}^T \quad (4)$$

$$u_j = \begin{bmatrix} u_j[0] & u_j[1] & \cdots & u_j[N] \end{bmatrix}^T \quad (5)$$

where $y_j$ and $u_j$ are the vector. Then, it is assumed that the transfer function $G[q]$ is causal system as follows.

$$G[q] = g_0 q^{-1} + g_1 q^{-2} + g_2 q^{-3} + \cdots \quad (6)$$

Here, $g_k$ is the impulse response of $G[z]$ at time $t = kT_s$ ($k = 0, 1, \cdots$). Thus, transfer function $G$ can represent by

$$G = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_N & g_{N-1} & \cdots & g_0 \end{bmatrix} \quad (7)$$

From above equation, the relationship eq.(3) is shown by $y_j = Gu_j$. where $G$ is Toeplitz and triangle matrix. Thus, the product of the matrices written in the form of eq.(7) becomes commutative. In this section, the stability of ILC is examined using the form of eq.(5) (7).

B. Without ILC

It is assume that the iterative reference trajectory $r$ which period is $T_{ite}$. Then, the output and the tracking error are $y_j$ and $e_i = r - y_j$ which is represented by

$$e_j = r - Pu_j = r - P_{fb} e_j$$  

$$= (I + P_{fb})^{-1} r = S_{fb} r \quad (8)$$

where $I$, $P$, $P_{fb}$, $S_{fb}$ are the unit matrix, the plant, the feedback controller, and the sensitivity function, respectively. Thus, the tracking error $e_j$ in iteration operation is only determined by the sensitivity function $S_{fb}$.

C. Iterative learning control system

There are some methods of ILC. In this paper, the control system adopted Previous Current Cycle Learning (PCCL) which is combined Current Cycle Learning and Previous Cycle Learning. Thus, PCCL has both characteristic [10], also PCCL is one of the most popular type of ILC. The equation of data renewal of PCCL is represented by

$$u_{j+1} = Q(u_j + Le_j) + C_{fb} e_{j+1} \quad (9)$$

where $Q$ is the lowpass filter which adjusts the stability of the system. $L$ is the learning filter which is the inverse system of the plant, usually. The control block diagram is shown in Fig. 3. From eq. (9), the control input at the iteration number $j+1$ uses the signal of the one period before which is stored by the FIFO memory. The tracking error of eq.(9) is represented by

$$e_{j+1} = Q(I - PL) S_{fb} e_j + (I - Q)e_1 \quad (10)$$

The stability condition which is derived by eq.(10) is represented by

$$||Q(I - PL) S_{fb}||_\infty < 1. \quad (11)$$

Hence, if the infinity norm $||Q(I - PL) S_{fb}||_\infty$ is smaller than 1, the tracking error will be converged. Thus, if $Q$ and $L$ can design as

$$Q = I, \quad L = P^{-1} \quad (12)$$
the tracking error \( e_{j+1} \) can suppress to zero at the iteration number \( j = 2 \) \( (e_2 = 0) \). From above reason, the learning filter \( L \) is designed as the inverse system of the plant, usually. However, in actual systems, there is the modeling error between the plant and the nominal plant. Here, it is assume that \( P = P_n(I + \Delta) \) and \( L = P_n^{-1} \). Then, the tracking error is represented by

\[
e_{j+1} = Q\Delta S_{fb} e_j + (I - Q)e_1. \tag{13}
\]

Generally, the nominal plant is designed give priority from the low frequency domain and correspond with the characteristic of the plant. Therefore, the modeling error is the inclination which becomes large in the high frequency domain. Moreover, in the positioning control system, the steady-state deviation becomes zero, and the feedback band is extended as much as possible, giving the adequate stability margin. As the result, the infinite norm of the sensitivity function becomes larger than 1 from the integration theorem of Bodo in the high frequency domain. Therefore, \( Q \) filter has the desirable low pass characteristic, in order to suppress the gain of the high frequency domain of \( \Delta S_{fb} \) to less than 1. However, since the tracking error also increases as the high frequency domain is cut so that clearly from the second term of the right-hand side of the eq. (13), it is not desirable to enlarge the order of \( Q \) filter.

Then, the construction of PCCL is considered. First, the renewal type of data of the e. (9) is transformed such as the

\[
e_{j+1} = Q(u_j + P_n^{-1}(r - y_j)) + C_{fb}e_{j+1} = P_n^{-1}Qr + C_{fb}e_{j+1} + Q(u_j - P_n^{-1}y_j). \tag{14}
\]

From the eq.(eq:PCCL1), \( P_n^{-1}Qr \) is the feedforwar input which is generated by the reference, \( C_{fb}e_{j+1} \) is the control input which is generated by the feedback controller, \( u_j - P_n^{-1}y_j \) is the control input as the estimated input disturbance at the one period before. Thus, it becomes clear from the control structure of PCCL to suppress the disturbance and the modeling error by the periodic disturbance observer, and also to converge quickly by the feedforward input using the inverse system of the plant.

D. Feedback controller

The feedback controller \( C_{fb}(s) \) which is PI compensator is represented by

\[
C_{fb}(s) = 10 \times 10^{-3} \cdot \frac{s + 1.885 \times 10^8}{s}. \tag{15}
\]

In implementation, \( C_{fb}(s) \) is discretize by tusin at the sampling time \( T_s \).

E. Notch filter

To suppress the peak of the sensitivity function, the notch filter is inserted after the \( C_{fb} \). In ILC, the infinity norm of the control system have to keep less than 1 for the stable. The designable controller are the learning filter \( L, Q \) filter, and FB controller \( C_{fb} \). Although designing all three controllers collectively is also considered, the engineer wants to give

the physical meaning of each controller. If it thinks from the eq. (10), the modeling error will cause gain buildup from the first term of the right-hand side that \( L \) inserts the notch in the inverse model. \( Q \) it's more natural to insert in \( C_{fb} \). Here, when the notch characteristic is given to \( Q \), it has influence on the second term of the right-hand side. It is better to include the notch characteristic in the feedback controller. The notch filter which negates the resonance peak is designed. The notch frequency is 5.65 [kHz].

\[
C_{notch}(s) = \frac{s^2 + 7100s + 1.26 \times 10^9}{s^2 + 3.55 \times 10^4s + 1.26 \times 10^9} \tag{16}
\]

F. Design of learning filter

It explained using the inverse system of the plant for the learning filter. However, in the relative order, in the continuous time domain, the unstable-zero generates the 3rd more than plant by discretization. Therefore, the stable inverse system cannot be composed [11]. This paper compares the case where ZPETC, ZMETC, and PTC which compose the inverse system from the discrete domain are used for ILC.

1) Learning filter based on ZPETC: The inverse system of the plant derived by ZPETC[5] realizes the zero phase characteristic in all the frequencies below nyquist frequency, when the future value is given to the input. The discretized plant is represented by

\[
P[z] = \frac{B[z]}{A[z]} = \frac{B_u[z]B_n[z]}{A[z]}. \tag{17}
\]

The characteristic polynomial \( A[z] \) is the stable. Where \( B^{-}[z^{-1}] \) is the s-th order monic polynomial equation which include the unstable or limit of the stable zeros, and \( B^+[z^{-1}] \) is the \((m - v)th\) order monic polynomial equation which include the stable zeros.

\[
B_u[z] = b_{un}z^n + b_{u(n-1)}z^{n-1} + \cdots + b_{u0} \tag{18}
\]

where the \( B_u[z] \) is rewrited as

\[
B_u[z] = b_{u0}z^n + b_{u1}z^{n-1} + \cdots + b_{un}. \tag{19}
\]

Then, ZPETC based inverted model of the plant is obtained by

\[
G_{zpetc}[z] = \frac{z^{-q}A[z]B_u^*[z]}{B_u[z](B_u[1])^2} \tag{20}
\]

Fig. 4 shows the bode diagram of the nominal plant \( P_n[z] \) and \( G_{zpetc}[z] \). Although the phase characteristic corresponds, the error has arisen in the gain characteristic as nyquist frequency is approached.

2) Learning filter based on ZMETC: The inverse system of the plant derived by ZMETC[3][4] realizes the zero magnitude characteristic in all the frequencies below nyquist frequency. Then, ZMETC based inverted model of the plant is obtained by

\[
G_{zmet}[z] = \frac{z^{-q}A[z]B_u^*[z]}{B_u[z](B_u[1])^2} \tag{21}
\]

Fig. 5 shows the bode diagram of the nominal plant \( P_n[z] \) and \( G_{zmet}[z] \). Although the gain characteristic corresponds, the error has arisen in the phase characteristic as nyquist frequency is approached.
3) Learning filter based on PTC: Perfect Tracking Control (PTC) [8] is the control method which can achieve zero tracking error on the sample point to the nominal plant. The state equation which realized the controllable form is represented by

\[
\dot{x}(t) = A_x x(t) + b_x e(t),
\]

where \(x(t)\) is the state variable. Then, the plant is discretized. The relation of the control period is \(T_u = T_y\) and \(T_r = nT_u\), respectively. The state equation discretized by sampling-period \(T_u\) is

\[
x[k+1] = A_k x[k] + b_k e[k]
\]

Here \(x[k]\) shows \(x(kT_u)\).

\[
A_k = e^{A_x T_u}, \quad b_k = \int_0^{T_u} e^{A_x \tau} b_x d\tau
\]

Since the order of the plant is \(n\), the \(n\) sample future of the eq. (23) is considered. It is considered as time \(t = iT_r = kT_u\).

\[
x[i+1] = A_i x[i] + B_i e[i]
\]

where \(i = 0, 1, 2, \cdots, k\). The control input is obtained from eq.(25) as follows.

\[
u_{ptc}[i] = B_i^{-1}(I - z_r^{-1}A_i)e[i+1]
\]

Here, \(z_r^{-1}\) is the shift operator of sampling time \(T_r\). As for matrix \(B_i\), the holomorphy will be guaranteed if the plant is the controllable. Moreover, all the poles of the inverse system are at the origin of \(z\) plane. By the eq.(26), since the state variable was necessary, the state variable undetectable in this paper was derived by central difference. For example, \(\dot{e}[k]\) of deflection was derived by the following equation.

\[
\dot{e}[k] \approx \frac{y[k+1] - y[k-1]}{2T_u}
\]

G. Design of Q filter

Q filter is represented by

\[
Q[z] = \left(\frac{z^{-1} + 2 + z}{4}\right)^{N_q}
\]

The filter is the zero phase low pass filter, and cut-off frequency is not small according to the increase in \(N_q\). First, it is Fig. 6 about \(\triangle S_{fb}[z]\) in case the plant and the nominal plant are eqs. (1) (2), and the frequency response of the Q filter is shown.

As Fig. 4 and 5 showed, both of \(G_{zptc}[z]\) and \(G_{zmet}[z]\) should express the plant with sufficient accuracy, but Fig. 6 the big difference is seen in \(\triangle S_{fb}[z]\). In the case where \(G_{zptc}[z]\) is used, below in 0 [dB], the gain has become, and has brought the desirable result. On the other hand, when \(G_{zptc}[z]\) is used, the modeling error serves as the big gain in the high region. It can check that the modeling error is greatly concerned with stability from these results. Moreover, since the perfect inverse system can be composed from PTC, it can expect that the gain is falling rather than these two methods. Then, the design to the actual plant is considered. First, the frequency characteristics of sensitivity function \(S_{fb}[z]\) and modeling error \(\triangle\) are investigated. Since the modeling error \(\triangle\) into consideration, high order fitting model \(P_{nhi}(s)\) shown in the following equation is prepared.

\[
P_{nhi}(s) = \frac{180.438 \times 10^7}{s^2 + 2036s + 5.116 \times 10^8}
\]

\[
\times \frac{1.244 \times 10^8 + 3.198 \times 10^6}{s^2 + 2890s + 3.341 \times 10^9}
\]

\[
\times \frac{5.701 \times 10^8}{s^2 + 1910s + 5.701 \times 10^8}
\]

\[
\times \frac{3581s + 1.283 \times 10^9}{s^2 + 3581s + 1.283 \times 10^9}
\]

The frequency response at the time of making the plant into the eq. (29) is shown in Fig. 7. Although it is considered by Fig. 6 that \(N_q = 1\) is enough as the Q filter, if it Fig. 7 Sees, the gain of the high region is rising greatly under the effect of the modeling error. Since it is considered that there is the bigger modeling error than the actually result, it can be guessed that the Q filter about \(N_q = 2\) is needed. After designing the Q filter and the learning filter, memory read timing is determined according to each learning filter.
IV. SIMULATION AND EXPERIMENTAL RESULT

A. Simulation

Triangular wave trajectory is used for actuation of the direction of X with the piezo scanner of AFM. The reason for using the triangular wave is to keep the constant speed during the X scan. The frequency of the triangular wave was made into 1 [kHz]. The simulation result at then [this] is shown in Fig. 9. It is the result in (a) ZMETC, (b) ZPETC, and (c) PTC, respectively. It is the case where the plant and the nominal plant are made into the eq.(1) (2) (considerable in Fig. 6). Moreover, the \( Q \) filter is not used in order to clarify the difference in the learning filter. It is the tracking error for six periods, respectively. Since 1 period serves as the answer of only FB, its following nature is bad, but in (b) ZPETC and (c) PTC, the tracking error can be powerfully suppressed after 2 period, respectively. (b) Although the result with most sufficient ZPETC was brought, since it asked for the state variable \((\dot{e}, \ddot{e})\) undetectable in PTC by central difference, such the result was brought. It can check that the analyses in Fig. 6 are right.

B. Experimental result

The triangular wave trajectory of 1 [kHz] of the same conditions as simulation was given. The amplitude is 800.

It is [nm]. An experimental result is shown in Fig. 10 and 11. Respectively, they are the reference track, and the position and deflection. Deflection piles up the experimental result and average value of 100 periods. Convergence is dramatically fast even when which learning filter is used. However, since the order of the \( Q \) filter had become in \( N_q = 1 \) oscillation in \( G_{z\text{pet}}[z] \), it was considered as \( N_q = 2 \). Under this effect, deflection is large the little in \( G_{z\text{pet}}[z] \).

V. CONCLUSION

In this paper, ILC was applied to the piezo scanner and fast positioning was carried out. Moreover, ZPETC, ZMETC, and PTC were used for the learning filter of ILC, and comparison analyses were carried out. It analyzed about the stability of ILC and the adequate \( Q \) filter was designed from the stability condition. From simulation and the experimental result, it was checked that the learning filter based on ZPETC is effective. Although the learning filter based on PTC was considered to be the effective method, the effect by central difference was checked. Since the learning filter based on ZMETC has buildup of the gain in the high region, it is necessary to make the number of section of the \( Q \) filter increase as compared with the two previous methods. Fast operation of 1 [kHz] was achieved not only in simulation but in the experiment.

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REFERENCES

Fig. 8. Simulation results (tracking error).

(a) ZMETC based ILC
(b) ZPETC based ILC
(c) PTC based ILC

Fig. 9. Simulation results (tracking error).

(a) ZMETC based ILC
(b) ZPETC based ILC
(c) PTC based ILC

Fig. 10. Experimental results (tracking error).

(a) ZMETC based ILC
(b) ZPETC based ILC
(c) PTC based ILC

Fig. 11. Experimental results (Reference and Output).